Absorption in Solar Atmosphere

A black body spectrum emitted from solar surface causes excitation processes on atoms in the solar atmosphere. This in turn causes absorption of characteristic wavelengths corresponding to those atoms that are the most abundant ones in the solar atmosphere – absorption spectroscopy.
Solar Emittance Spectrum

Wavenumber:  \( k = \frac{v}{c} = \frac{1}{\lambda} \left[ cm^{-1} \right] \)

Spectrum is characterized by many atomic absorption lines, primarily by H-Balmer series of highly abundant hydrogen.
The Hydrogen Atom

http://www.walter-fendt.de/ph14e/bohrh.htm
http://www.falstad.com/qmatom/

\[ E_n = - Z^2 \frac{E_0}{n^2} \quad \text{with} \quad E_0 = 13.6 \text{ eV} \]

\[ E_0 \] is energy necessary to ionize atom

\[ Z \] is charge of nucleus, \( Z = 1 \)

\[ n = \text{main quantum number, } n=0, 1, 2, ... \]

\[ \ell = \text{orbital momentum quantum number, } \ell = 1, 2, 3, ...n-1 \]

\[ m = \text{magnetic quantum number, } 2\ell+1 \text{ values from } -\ell, -\ell+1 ...0... \ell+1, \ell \]

\[ s = \frac{1}{2} \text{ spin quantum number, } s=1/2, -1/2 \]

Electron transition between orbits requires or releases energy:
Transition between Energy Orbitals

Possible emission or absorption of light with fixed wavelengths by transitions of electrons between orbits! Wavelength depends on energy difference between orbits!

Hydrogen emission spectrum

$E_n = -13.6/n^2$

Balmer Series

$N=3 \rightarrow 2$

$H_\alpha$: $\lambda=656.3$nm

$n=1$

$n=2$

$n=3$, L shell

$n=4$, N shell

$n=5$

$n=6$

$N=3$, K shell

$N=4$, L shell

$N=5$, L shell
Energy and Wavelength of emitted or absorbed Photons

\[ E_v = E_{n_i} - E_{n_f} = 13.6 \cdot \left( \frac{1}{n_f^2} - \frac{1}{n_i^2} \right) [eV] \]

\[ E_v = h \cdot \nu = \frac{h \cdot c}{\lambda} \quad \frac{1}{\lambda} = \frac{13.6}{h \cdot c} \cdot \left( \frac{1}{n_f^2} - \frac{1}{n_i^2} \right) [eV] \]

\[ h \equiv \text{Planck Constant: } 6.626 \cdot 10^{-34} [J \cdot s] \]

\[ c \equiv \text{Speed of Light: } 2.987 \cdot 10^8 [m/s] \]

\[ \frac{13.6 [eV]}{c \cdot h} = 1.097 \cdot 10^7 \left[ \frac{1}{m} \right] = R_H \]
Multi-Electron Atoms

$Z$ is the charge and determines the chemical characteristics of the element, e.g. $Z=1$ (Hydrogen), $Z=2$ (Helium) $Z=8$ (Oxygen).

Electron transitions and photon emission is more complex and depends on $Z$ and the shielding $S_n$ by inner electron shells.

\[ E_n = -(Z - S_n)^2 \cdot \frac{13.6}{n^2} \approx -(Z - 1)^2 \cdot \frac{13.6}{n^2} \]

\[ E_K = E_n - E_1 = -(Z - 1)^2 \cdot \frac{13.6}{n^2} + (Z - 1)^2 \cdot \frac{13.6}{1^2} \]

\[ E_K = (Z - 1)^2 \cdot 13.6 \cdot \left(1 - \frac{1}{n^2}\right) = \frac{hc}{\lambda_K} \]

\[ \lambda_K = \frac{hc}{E_\gamma} = \frac{hc}{E_n - E_1} \]
Example: Calculate the K and L UV-Ray Lines for He

\[ E_K = (Z - 1)^2 \cdot 13.6[\text{eV}] \cdot \left(1 - \frac{1}{n_i^2}\right) \]

**He:** \( Z = 2 \)

\[ E_{K\alpha} = 1 \cdot 13.6[\text{eV}] \cdot \left(1 - \frac{1}{4}\right) = 10.2[\text{eV}] \quad \lambda = \frac{h \cdot c}{E_{K\alpha}} = 121.6\text{nm} \]

\[ E_{K\beta} = 1 \cdot 13.6[\text{eV}] \cdot \left(1 - \frac{1}{9}\right) = 12.09[\text{eV}] \quad \lambda = \frac{h \cdot c}{E_{K\beta}} = 102.6\text{nm} \]

\[ E_L = (Z - 1)^2 \cdot 13.6[\text{eV}] \cdot \left(\frac{1}{2^2} - \frac{1}{n_i^2}\right) \]

\[ E_{L\alpha} = 1 \cdot 13.6[\text{eV}] \cdot \left(1 - \frac{1}{4} - \frac{1}{9}\right) = 1.889[\text{eV}] \quad \lambda = \frac{h \cdot c}{E_{L\alpha}} = 656.8\text{nm} \]

**K\alpha** and **K\beta** transitions in the ultraviolet range!

**L\alpha** and other L and M transitions in visible range.
Each element has its own characteristic transitions depending on the orbit quantum numbers and the charge Z (number of electrons, protons). These transitions can be analyzed by spectroscopy of the light to determine the elemental abundance!
Absorption in the Earth Atmosphere

Absorption and scattering of radiation in the atmosphere depends on the chemical and physical composition of the atmospheric layers. This includes dust and molecular composition, moisture, temperature as well as density. It also depends on the nature of the interaction processes of the incoming (and exiting) radiation (photons) with the atomic and molecular gas components!
Solar Radiation in Atmosphere

\[ I = I_0 \cdot e^{-\mu \cdot d} \]

\[ F(\lambda) = F_0(\lambda) \cdot e^{-\int \mu(\lambda, x) \cdot dx} \]

- \( F(\lambda) \): transmitted radiation
- \( \mu(\lambda, x) \): absorption coefficient
- \( x \): thickness of atmosphere layer

The absorption coefficient includes absorption probabilities by molecular rotational or vibrational excitation processes, and photon scattering, it depends on the composition of the atmosphere and its atomic and molecular components: \( N_2, O_2, O_3, CO_2, CH_4, H_2O \)
Absorption Probabilities

Transmission

\[
\frac{F(\lambda)}{F_0(\lambda)} = e^{-\int_{x}^{x} \mu(\lambda, x) \cdot dx}
\]

\[
\mu(\lambda, x) = \sum_{i} n_i(x) \cdot \sigma_i(\lambda)
\]

\[
n_i = \rho(x) \cdot \frac{X_i(x)}{A_i} \cdot N_A
\]

The particle density \( n_i(x) \) depends on the overall radial density dependence and the mass fractions of the gas components in the different atmospheric layers.

Absorption (heat production in absorbing material)

\[
\frac{F_{abs}(\lambda)}{F_0} = 1 - \frac{F(\lambda)}{F_0} = 1 - (\lambda) \cdot e^{-\int_{x}^{x} \mu(\lambda, x) \cdot dx}
\]

\( n_i(x) \): particle density (cm\(^{-3}\))

\( \sigma_i(\lambda) \): cross section (cm\(^2\))

The cross section \( \sigma_i \) is the interaction probability of light (photons) with a certain wavelength \( \lambda \) or energy \( E=h \cdot \nu \) with various atmospheric gases and dust particles (excitation and scattering)
# Atmospheric Composition

<table>
<thead>
<tr>
<th>Gases</th>
<th>% by volume</th>
<th>Comments</th>
</tr>
</thead>
<tbody>
<tr>
<td>Nitrogen, N₂</td>
<td>78.08%</td>
<td>Photochemical dissociation high in the ionosphere; mixed at lower levels</td>
</tr>
<tr>
<td>Oxygen, O₂</td>
<td>20.95%</td>
<td>Photochemical dissociation above 95 km; mixed at lower levels</td>
</tr>
<tr>
<td>Argon, Ar</td>
<td>0.93%</td>
<td>Mixed up to 110 km</td>
</tr>
<tr>
<td>Neon, Ne</td>
<td>0.0018%</td>
<td></td>
</tr>
<tr>
<td>Helium, He</td>
<td>0.0005%</td>
<td>Mixed in most of the middle atmosphere</td>
</tr>
<tr>
<td>Krypton, Kr</td>
<td>0.00011%</td>
<td></td>
</tr>
<tr>
<td>Xenon, Xe</td>
<td>0.000009%</td>
<td></td>
</tr>
</tbody>
</table>

## Variable gases

<table>
<thead>
<tr>
<th>Gases</th>
<th>% by volume</th>
<th>Comments</th>
</tr>
</thead>
<tbody>
<tr>
<td>Water vapor, H₂O</td>
<td>4.0% (maximum, in the tropics) 0.00001% (minimum, at the South Pole)</td>
<td>Highly variable; photodissociates above 80 km dissociation</td>
</tr>
<tr>
<td>Carbon dioxide, CO₂</td>
<td>0.0365% (increasing ~0.4% per year)</td>
<td>Slightly variable; mixed up to 100 km; photodissociates above</td>
</tr>
<tr>
<td>Methane, CH₄</td>
<td>~0.00018% (increases due to agriculture)</td>
<td>Mixed in troposphere; dissociates in mesosphere</td>
</tr>
<tr>
<td>Hydrogen, H₂</td>
<td>~0.00006%</td>
<td>Variable photochemical product; decreases slightly with height in the middle atmosphere</td>
</tr>
<tr>
<td>Nitrous oxide, N₂O</td>
<td>~0.00003%</td>
<td>Slightly variable at surface; dissociates in stratosphere and mesosphere</td>
</tr>
<tr>
<td>Carbon monoxide, CO</td>
<td>~0.00009%</td>
<td>Variable</td>
</tr>
<tr>
<td>Ozone, O₃</td>
<td>~0.000001% - 0.0004%</td>
<td>Highly variable; photochemical origin</td>
</tr>
<tr>
<td>Fluorocarbon 12, CF₂Cl₂</td>
<td>~0.00000005%</td>
<td>Mixed in troposphere; dissociates in stratosphere</td>
</tr>
</tbody>
</table>
Large variation in $n_i(x)$ with altitude for different gaseous elements and molecules. Enrichment of H$_2$O vapor in lower atmosphere, the troposphere. CO$_2$, O$_2$, Ar fairly continuous, O$_3$ break up in stratosphere!
Rayleigh Scattering

Rayleigh scattering is the elastic scattering of light by particles of size $d$ much smaller than the wavelength of the light $\lambda$, such as molecules or dust particles. Rayleigh scattering is a function of the electric polarizability of the particles.

$$d \ll \lambda \quad d \leq 100 \text{ nm}$$

$$\sigma(\lambda) \approx 44.7 \cdot \pi^5 \cdot \frac{\alpha^2}{\lambda^4}$$

Volume polarizability: $\alpha_{gas} = 1.7 \cdot 10^{-24} \text{ cm}^3$

Scattering cross section increases with lower wavelengths, blue light gets more scattered than red light. It gives the sky its typically blue color (otherwise sky would have solar white spectrum color). Looking at sunset, scatter has increased because of increased thickness of atmospheric layer, blue is removed from line of sight to sun.
Example

Calculate Rayleigh scattering cross section on air for blue \( \lambda_b = 450 \text{ nm} \) and red light \( \lambda_r = 650 \text{ nm} \); \( \alpha_{gas} = 1.7 \cdot 10^{-24} \text{ cm}^3 \)

\[
\sigma(\lambda) \approx 44.7 \cdot \pi^5 \frac{\alpha^2}{\lambda^4} = \frac{3.953 \cdot 10^{-44}}{(\lambda [cm])^4}
\]

\[
\sigma(\lambda_b) \approx 9.64 \cdot 10^{-27} \text{ cm}^2 \\
\sigma(\lambda_b) \approx 9.64 \text{ mbarn} \\
\sigma(\lambda_r) \approx 2.22 \cdot 10^{-27} \text{ cm}^2 \\
\sigma(\lambda_3) \approx 2.22 \text{ mbarn}
\]

\( 1 \text{ barn} = 10^{-24} \text{ cm}^2 \)
\[ \mu(\lambda, x) = \sum_i n_i(x) \cdot \sigma_i(\lambda) \quad n_i = \rho(x) \cdot \frac{X_i(x)}{A_i} \cdot N_A \]

Wave length \( \lambda = 550 \text{ nm} \), Density \( \rho \approx 1 \text{ g/cm}^3 \), Thickness of atmosphere layer \( d \approx 10 \text{ km} \)

<table>
<thead>
<tr>
<th></th>
<th>( x_i )</th>
<th>( \alpha_i )</th>
<th>( A_i )</th>
<th>( n_i \text{[cm}^{-3}\text{]} )</th>
<th>( \sigma_i \text{[cm}^2\text{]} )</th>
<th>( \mu_i \text{[cm}^{-1}\text{]} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \text{N}_2 )</td>
<td>0.78</td>
<td>1.77</td>
<td>28</td>
<td>1.6776E+22</td>
<td>4.67145E-27</td>
<td>7.8366E-05</td>
</tr>
<tr>
<td>( \text{O}_2 )</td>
<td>0.19</td>
<td>0.8</td>
<td>32</td>
<td>3.5756E+21</td>
<td>9.543E-28</td>
<td>3.4122E-06</td>
</tr>
<tr>
<td>( \text{Ar} )</td>
<td>0.01</td>
<td>1.66</td>
<td>38</td>
<td>1.5847E+20</td>
<td>4.10886E-27</td>
<td>6.5115E-07</td>
</tr>
<tr>
<td>( \text{H}_2\text{O} )</td>
<td>0.01</td>
<td>1.48</td>
<td>18</td>
<td>3.3456E+20</td>
<td>3.26609E-27</td>
<td>1.0927E-06</td>
</tr>
<tr>
<td>( \text{CO}_2 )</td>
<td>0.003</td>
<td>2.63</td>
<td>44</td>
<td>4.1059E+19</td>
<td>1.03137E-26</td>
<td>4.2347E-07</td>
</tr>
<tr>
<td>( \text{SO}_4 )</td>
<td>0.001</td>
<td>4.34</td>
<td>98</td>
<td>6.1449E+18</td>
<td>2.80856E-26</td>
<td>1.7258E-07</td>
</tr>
<tr>
<td>Air</td>
<td>0.994</td>
<td>1.7</td>
<td>2</td>
<td>2.0891E+22</td>
<td>4.30926E-27</td>
<td>9.0026E-05</td>
</tr>
</tbody>
</table>

\[
\left( \frac{F(\lambda)}{F_0(\lambda)} \right)_{\text{air}} = e^{-\int \mu(\lambda, x) \cdot dx} \approx e^{-\mu \cdot d} \approx e^{-9.0 \cdot 10^{-5} \cdot 10^5} = 1.23 \cdot 10^{-4} \quad \text{everything scattered}
\]

\[
\left( \frac{F(\lambda)}{F_0(\lambda)} \right)_{\text{H}_2\text{O}} = e^{-\int \mu(\lambda, x) \cdot dx} \approx e^{-\mu \cdot d} \approx e^{-1.1 \cdot 10^{-6} \cdot 10^5} = 3.33 \cdot 10^{-1} \quad 66\% \text{ scattered}
\]
Rayleigh Scattering

\[ \sigma(k) \approx 44.7 \cdot \pi^5 \cdot \alpha^2 \cdot k^4 \]

Transmission through 10 km air (vertically)

Transmission through 30 km air (horizontally)

Scattering cross section in barn

\[ \left( \frac{F(\lambda)}{F_0(\lambda)} \right)_{\text{air}} = e^{-\int \mu(\lambda,x) \cdot dx} \approx e^{-\mu \cdot d} \]