Earth energy budget and balance II
Emissivity

Absorbed incoming radiation flux heats up surface material, which in turn emits thermal radiation. The **emissivity** of a surface is its effectiveness in emitting its energy as thermal radiation into the outer space!

\[ F_e = F_t + F_a \]

\[ 1 = \frac{F_t}{F_e} + \frac{F_a}{F_e} = \varepsilon + \frac{F_a}{F_e} \]

\[ \varepsilon = 1 - \frac{F_a}{F_e} \]

As higher the absorption for emitted radiation in atmosphere as lower is the emissivity \( \varepsilon \)

No absorption: \( \varepsilon = 1 \) (black body)

We ignore reflection back
Emissivity

Emissivity, ε, is defined as the ratio of the emitted radiation at a specific wavelength and temperature to the emitted radiation from a black body at the same wavelength and temperature.

\[ F_\lambda = \varepsilon_\lambda \cdot B_\lambda(T_s) \]

Flux in a fixed wavelength range

Emissivity in IR range \( \varepsilon_\lambda > 0.8 \) 
(\( \lambda = 4\mu m \ – 100\mu m \))

For \( \lambda = 8\mu m \ – 12\mu m \)

<table>
<thead>
<tr>
<th>Surface</th>
<th>Emissivity</th>
</tr>
</thead>
<tbody>
<tr>
<td>Water</td>
<td>0.993-0.998</td>
</tr>
<tr>
<td>Ice</td>
<td>0.98</td>
</tr>
<tr>
<td>Green grass</td>
<td>0.975-0.986</td>
</tr>
<tr>
<td>Sand</td>
<td>0.949-0.962</td>
</tr>
<tr>
<td>Snow</td>
<td>0.969-0.997</td>
</tr>
<tr>
<td>Granite</td>
<td>0.898</td>
</tr>
<tr>
<td>White paint</td>
<td>0.60-0.75</td>
</tr>
<tr>
<td>Black paint, tar</td>
<td>0.9-1.0</td>
</tr>
</tbody>
</table>
Day Night Emissivity

More emission during evening night, when earth is cooling
Wave length dependence of emissivity

Emissivity spectra are a function of surface types including water, old snow, fresh snow, soil, tilled soil, sand, rock, irrigated low vegetation, meadow grass, scrub, broadleaf forest, pine forest, tundra, grass soil, broadleaf pine forest, grass scrub, oil grass, urban concrete, pine brush, broadleaf brush, wet soil, scrub soil, broadleaf 70-pine 30, and new ice.
Uncertainty range of emissivity predictions for remote sensing

William G. Snyder et al. 2005
Impact of emissivity on equilibrium temperature

\[ F_\lambda = \varepsilon_\lambda \cdot B_\lambda(T_s) \]

\[ F_{\text{emitted}} = \varepsilon \cdot \sigma \cdot T^4, \quad \varepsilon \leq 1 \]

\[ S = 4\pi \cdot R^2 \cdot F \]

Averaging over the wavelength dependence:

\[ S_{\text{emission}} = S_{\text{absorption}} \quad \Leftrightarrow \quad \text{approximate assumption} \]

\[ S_{\text{absorption}} = (1 - \alpha) \cdot S_0 = (1 - \alpha) \cdot \pi \cdot R^2 \cdot F_0 \]

\[ S_{\text{emission}} = 4\pi \cdot R^2 \cdot \varepsilon \cdot \sigma \cdot T_{\text{emission}}^4 \]

\[ (1 - \alpha) \cdot F_0 = 4 \cdot \varepsilon \cdot \sigma \cdot T_{\text{emission}}^4 \]

\[ T_{\text{emission}} = \sqrt[4]{\frac{(1 - \alpha) \cdot F_0}{4\varepsilon \cdot \sigma}} \]
Emission temperature of dark asphalt on a sunny summer day

\[ T_{\text{asphalt}} = \left( \frac{(1-\alpha) \cdot F_0}{4 \cdot \varepsilon \cdot \sigma} \right)^{\frac{1}{4}} \]

\[ F_0 = 1.37 \cdot 10^3 \frac{W}{m^2} \]

\[ \sigma = 5.67 \cdot 10^{-8} \text{ W m}^{-2} \text{ K}^{-4} \]

Asphalt surface or black tar roof; \( \alpha = 0.1, \varepsilon = 0.97 \)

\[ T_{\text{asphalt}} = 274 K \]

\[ F_{\text{asphalt}} = \sigma \cdot \varepsilon \cdot T^4 = 308 \frac{W}{m^2} \]

White roof top; \( \alpha = 0.8, \varepsilon = 0.68 \)

\[ T_{\text{white}} = 205 K \]

\[ F_{\text{white}} = \sigma \cdot \varepsilon \cdot T^4 = 68.5 \frac{W}{m^2} \]
“The picture above depicts a hot summer day in California. The roof pictured is a Duro-Last roof. The yellow, orange and red colors are the areas of the highest temperatures in the photo (blue and violet correspond to cooler environments). Compare the roof of the building to the left of the Duro-Last roof. Notice it's colors are yellow, orange and red, while the Duro-Last roof colors are primarily blue.” Compare also the temperature range of the trees, it beats Duro-last!
Wavelength averaged values of albedo: $\alpha=31\%$
Wavelength averaged value of emissivity: $\varepsilon=80\%$

\[
T_{\text{emission}} = \sqrt[4]{\frac{(1 - \alpha) \cdot F_0}{4\varepsilon \cdot \sigma}}
\]

\[
T_{\text{emission}} = \sqrt[4]{\frac{(1 - 0.31) \cdot 1370 \frac{W}{m^2}}{4 \cdot 0.8 \cdot 5.67 \cdot 10^{-8} \frac{J}{K^4 m^2 s}}}
= 270 K
\]

Closer to average temperature of $T=280 \, K$ but still cold

A change of the solar constant by 1.4% or the average albedo 3.3%, or the average emissivity by 1.4% will change the average temperature by $1^\circ C$ !!!
Maintaining equilibrium

Constant temperature requires balance between absorption and emission of energy. Changing ratio will have consequences for local and possibly global temperature conditions. Most discussed change is CO$_2$ increase, which increases the absorption of emitted infrared radiation in the earth atmosphere, reducing the emissivity!
Impact of $\text{CO}_2$ abundance

http://climatemodels.uchicago.edu/modtran/
Absorption in the atmosphere

Infrared radiation from the earth surface of temperature $T_s$ is partially absorbed in atmosphere, generating a temperature $T_A$. Atmosphere is considered opaque to infrared but transparent to other wavelength.

Incoming net solar flux:

$$ F_{in} = \frac{(1-\alpha)F_0}{4} $$

Flux $A_\uparrow$ radiated to space depends on atmospheric temperature $T_a$:

$$ A_\uparrow = \varepsilon_a \cdot \sigma \cdot T_a^4 = A_\uparrow $$

Flux $F_\uparrow$ radiated from surface with terrestrial temperature $T_s$:

$$ F_\uparrow = \varepsilon_s \cdot \sigma \cdot T_s^4 $$

For thermal equilibrium:

$$ F_\uparrow = F_{in} + A_\downarrow \quad \text{net flux at ground} = 0 $$
Greenhouse effect

The thermal equilibrium requires:

\[ F_{\uparrow} - F_{\text{in}} - A_{\downarrow} = 0 \]
\[ F_{\uparrow} = F_{\text{in}} + A_{\downarrow} \]

\[ \sigma \cdot \varepsilon_s \cdot T_s^4 = \frac{(1 - \alpha) \cdot F_0}{4} + \sigma \cdot \varepsilon_a \cdot T_a^4 = 2 \cdot \sigma \cdot \varepsilon_a \cdot T_a^4 \]

\[ \Rightarrow T_s = 4 \sqrt{2 \cdot \frac{\varepsilon_a}{\varepsilon_s} \cdot T_a} \approx 1.19 \cdot T_a \]

\[ (\text{assuming } \varepsilon_s \approx \varepsilon_a) \]

With the calculated emission temperature \( T_a = 255K \) (without emissivity corrections) we obtain a increased surface temperature of \( T_s = 303K \). Overshooting, model is too simplified, more later!
CO$_2$ Impact on energy budget

<table>
<thead>
<tr>
<th>If the Earth has these properties...</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Direct Sunlight (W/m$^2$)</td>
<td>1366</td>
</tr>
<tr>
<td>Surface Albedo</td>
<td>Custom albedo</td>
</tr>
<tr>
<td>Albedo (fraction)</td>
<td>0.3</td>
</tr>
<tr>
<td>Surface Temp (K)</td>
<td>284.5</td>
</tr>
<tr>
<td>Lapse Rate (K/km)</td>
<td>6</td>
</tr>
<tr>
<td>Stratospheric Height (km)</td>
<td>15</td>
</tr>
<tr>
<td>CO$_2$ (ppm)</td>
<td>350</td>
</tr>
<tr>
<td>CH$_4$ (ppm)</td>
<td>1.7</td>
</tr>
<tr>
<td>Relative Humidity (%)</td>
<td>80</td>
</tr>
<tr>
<td>Low Cloud (fraction)</td>
<td>0</td>
</tr>
<tr>
<td>High Cloud (fraction)</td>
<td>0</td>
</tr>
<tr>
<td>Drop radius (10$^{-6}$m)</td>
<td>10</td>
</tr>
<tr>
<td>No aerosols</td>
<td>▼</td>
</tr>
</tbody>
</table>

...then it loses as much energy as it gains.

http://climatemodels.uchicago.edu/rrtm/