Homework 5 Clouds and Aerosols

1. The oceans contain 97% of the world’s water supply. The total water surface of the oceans is 361 Million km$^2$ and the average depth is about 3790 m. Only 2.25% of the water is stored in polar ice. Assume that 10% of the ice shelf melts in the next two decades, what would be the raise in sea level?

2. Assume a potential temperature of $\theta = 323$ K. Calculate the temperature in a rising cloud element at altitudes of $z=1000$ m and $z=5000$ m. Use the barometric formula for calculating the pressure $P$ at these altitudes and a scaled height of $H=7.31km$.

3. Sulfur dioxide is a noticeable component in the atmosphere, especially following volcanic eruptions. Since 1970 about 331,679,000 of tons of SO$_2$ have been released into the troposphere. Calculate the change of molecular weight of the troposphere since 1970 assuming an overall volume of $8.1 \cdot 10^{18}$ m$^3$. Calculate the percent volume of SO$_2$ assuming a homogeneous distribution in the troposphere.

4. Assume a dry day with 40% and a hot and humid day with 90% humidity, calculate the mass mixing ratio of water and air and determine the pressure of water vapor for both of these conditions.

5. Consider a NaCl salt particle of $10^{-19}$ kg in water vapor of U=90% and 100.2% relative humidity. Calculate the size of a water droplet condensing on the salt particle using the Kohler formula.
Solution Problem 5.1: The oceans contain 97% of the world’s water supply. The total water surface of the oceans is 361 Million km$^2$ and the average depth is about 3790 m. Only 2.25% of the water is stored in polar ice. Assume that 10% of the ice shelf melts in the next two decades, what would be the raise in sea level?

$$V_{melt} = 0.00225 \cdot V_o = 0.000225 \cdot 361 \cdot 10^6 \ km^2 \cdot 3.790 \ km$$

$$V_{melt} = 3078427.5 \ km^3 = 3.08 \cdot 10^6 \ km^3$$

$$H = \frac{V_{melt}}{S} = \frac{3.08 \cdot 10^6 \ km^3}{3.61 \cdot 10^8 \ km^2} = 8.5 \ m$$
Solution Problem 5.2: Assume a potential temperature of $\theta = 323$ K. Calculate the temperature in a rising cloud element at altitudes of $z=1000 \text{ m}$ and $z=5000 \text{ m}$. Use the barometric formula for calculating the pressure $P$ at these altitudes and a scaled height of $H=7.31 \text{ km}$.

\[
P = P_0 \cdot e^{-\frac{z}{H}}
\]

\[
P_{1000m} = 1 \text{ atm} \cdot e^{-\frac{1000}{7310}} = 0.872 \text{ atm}
\]

\[
P_{5000m} = 1 \text{ atm} \cdot e^{-\frac{5000}{7310}} = 0.504 \text{ atm}
\]

\[
T = \theta \cdot \left( \frac{P}{P_0} \right)^{\kappa}
\]

\[
T_{1000m} = 323K \cdot \left( \frac{0.872}{1} \right)^{2/7} = 311K
\]

\[
T_{5000m} = 323K \cdot \left( \frac{0.504}{1} \right)^{2/7} = 266K
\]
Solution Problem 5.3: Sulfur dioxide is a noticeable component in the atmosphere, especially following volcanic eruptions. Since 1970 about 331,679,000 of tons of SO₂ have been released into the troposphere. Calculate the change of molecular weight of the troposphere since 1970, assuming an overall volume of \(8.1 \cdot 10^{18} \text{ m}^3\). Calculate the percent volume of SO₂ assuming a homogeneous distribution in the troposphere.

Present molecular weight of air is: \(\bar{m}_{\text{air}} \approx 0.78 \cdot 28 + 0.21 \cdot 32 \approx 29\)

\[
\frac{n_{\text{air}}}{V} = \frac{\rho_{\text{air}} \cdot NA}{\bar{m}_{\text{air}}} \approx 500 \frac{g}{m^3} \cdot \frac{6.022 \cdot 10^{23}}{29} = 10^{25} \frac{\text{part}}{m^3}
\]

Total number of air particles:

\[
n_{\text{air}} = 10^{25} \frac{\text{part}}{m^3} \cdot 8.1 \cdot 10^{18} m^3 = 8.4 \cdot 10^{43} \text{ part}
\]

\[m_{\text{SO}_2} \approx 64 \text{ g / mole}\]

\[
n_{\text{SO}_2} = \frac{M}{m_{\text{SO}_2}} \cdot N_A \approx \frac{3.31679 \cdot 10^{14} \text{ g}}{64 \text{ g / mol}} \cdot 6.022 \cdot 10^{23} \frac{\text{part}}{\text{mol}}
\]

Total number of SO₂ particles:

\[n_{\text{SO}_2} = 3.1 \cdot 10^{36} \text{ part}\]

\[
\frac{n_{\text{SO}_2}}{n_{\text{air}}} = 3.7 \cdot 10^{-8} \quad \text{negligible in weight!}
\]
Solution Problem 5.4: Assume a dry day with 40% and a hot and humid day with 90% humidity, calculate the mass mixing ratio of water and air and determine the pressure of water vapor for both of these conditions.

\[ U = \frac{P_{H_2O}}{SVP} = 0.4, 0.9 \]

\[ x_m = \frac{0.662 \cdot U \cdot SVP}{P - (U \cdot SVP)} = \frac{0.662 \cdot U \cdot SVP}{1000 - (U \cdot SVP)} \]

\[ SVP = A \cdot e^{\beta \cdot T} \]

\[ A = 6.11 \text{ mbar} \]

\[ \beta = 0.067 \text{ }^\circ \text{C}^{-1} \]
Solution Problem 5.5:

Consider a NaCl salt particle of $10^{-19}$ kg in water vapor of $U=90\%$ and $100.2\%$ relative humidity. Calculate the size of a water droplet condensing on the salt particle using the Kohler formula.

$$U \approx 1 + \frac{a}{r} - \frac{b}{r^3}$$

$$a = 6 \cdot 10^{-3} \mu m \quad b = 2 \cdot 10^{-5} \mu m^2$$

$$r^* = \sqrt{\frac{3b}{a}} = \sqrt{\frac{3 \cdot 2 \cdot 10^{-5}}{6 \cdot 10^{-3}}} \mu m = 0.1 \mu m$$

$$U^* = 1 + \sqrt{\frac{4 \cdot a^3}{27 \cdot b}} = 1 + \sqrt{\frac{4 \cdot 2.16 \cdot 10^{-7}}{27 \cdot 2 \cdot 10^{-5}}} = 1.002$$

For a relative humidity of $100.2\%$ a solution droplet of 0.1$\mu$m would form

For a relative humidity of $90\%$ a solution droplet of 0.05$\mu$m would form