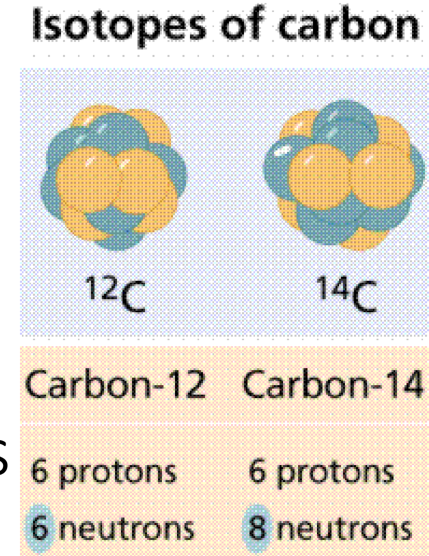
# Radioactivity

Lecture 5

The Nature and Laws of Radioactivity

# Changing Z to N or N to Z




## Adding a proton (electron)

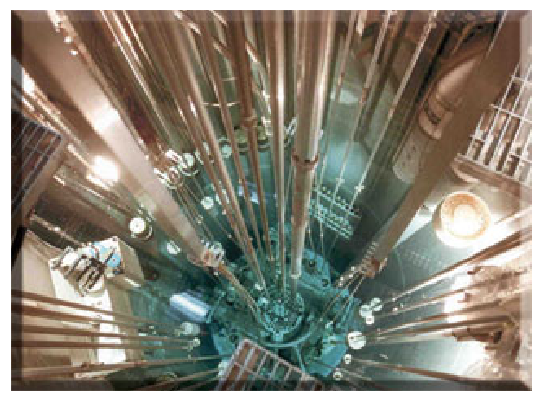


Carbon to Nitrogen


Gold to Mercury


## Subtracting or adding neutrons

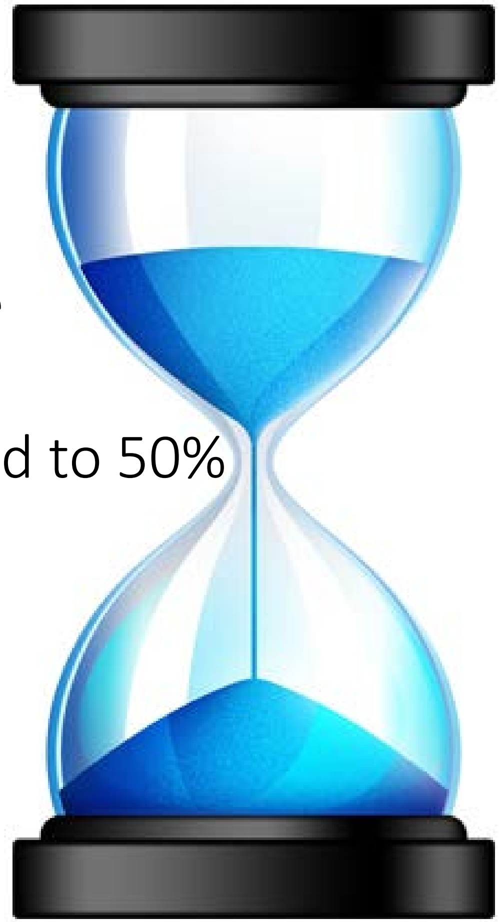
nucleus becomes unstable and decays by internally converting neutrons to protons (beta-decay)!



What are the physical laws that govern the decay process?

# Terminology of nuclear decay

Time dependent change
from configuration 1 (radioactive nucleus)
To configuration 2 (decay product, daughter)



- Activity: number of decay events per time
- Decay constant: probability of decay
- Half life: time for the activity to be reduced to 50%

- Activity corresponds to the number of sand particles dripping through hole
- Decay constant is associated with the size of the hole


# Units for the Activity A for a radioactive nucleus

- Classical unit: Curie: Ci corresponds to the number of decays of 1 g Radium as introduced by Madame Curie
- Modern unit: Becquerel: Bq notes a single decay event

$$1\ \text{Ci} = 3.7\cdot10^{10}\ \text{decays/s} = 3.7\cdot10^{10}\ \text{Bq}$$

$$1\ \text{Bq} = 1\ \text{decay/s}$$

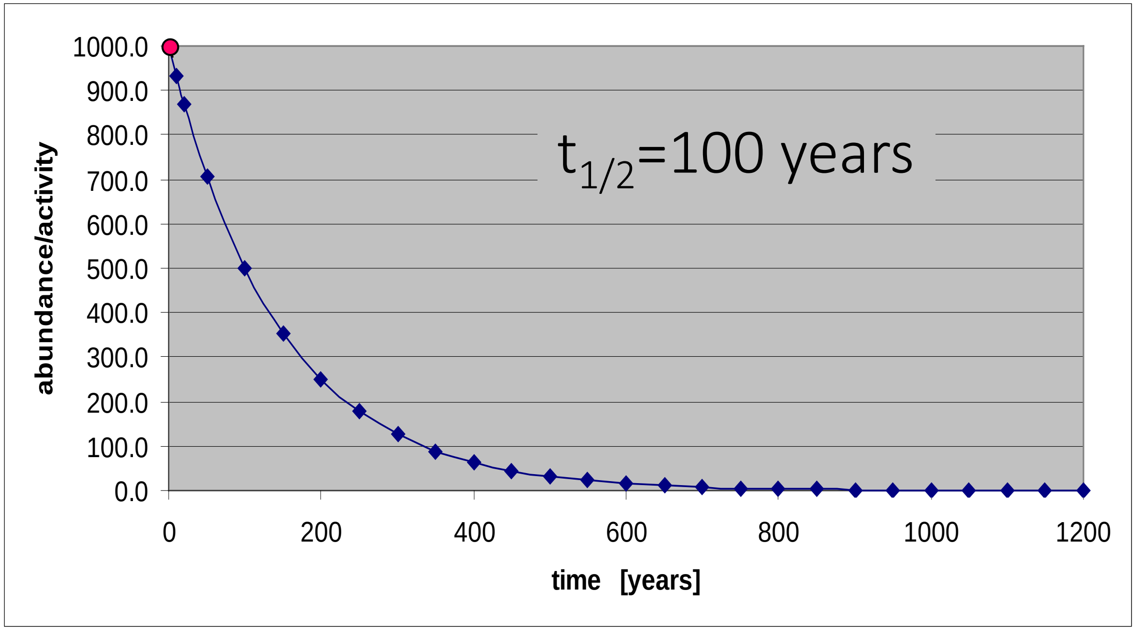
Example: the human body is radioactive with an activity of:

$2.2\cdot10^{-7}\text{Ci} = 0.22\mu\text{Ci}$ $\Rightarrow$ $8000\ \text{Bq} = 8\ \text{kBq}$

Sounds comfortably low — sounds alarmingly high

# Radioactive Decay Law

Describes the change of activity with time





$$A_{mother}(t) = A_0 \cdot e^{-\lambda \cdot t}$$

$$A_{daughter}(t) = A_0 \cdot \left(1 - e^{-\lambda \cdot t}\right)$$

exponential decay with time!
At half life 50% of the activity is gone!

λ≡decay constant;
a natural constant
for each radioactive
element.
Half life: $t_{1/2} = \ln 2/\lambda$

# $1^{st}$ example: $^{22}Na$

$^{22}Na$ is a radioactive nucleus with a half-life of 2.6 years, what is the decay constant?
Mass number A=22; (don't confuse with activity *A(t)!)*

$$\lambda = \frac{\ln 2}{t_{1/2}} = \frac{\ln 2}{2.6\, y} = 0.27\, y^{-1} :$$

$$1\, y = 3.14 \cdot 10^{7}\, s \approx \pi \cdot 10^{7}\, s$$

$$\lambda = \frac{\ln 2}{2.6 \cdot 3.14 \cdot 10^{7}\, s} = 8.5 \cdot 10^{-9}\, s^{-1}$$

# Radioactive Decay Laws

Activity of radioactive substance *A(t)* is at any time t proportional to number of radioactive particles *N(t)* :

$$A(t) = \lambda \cdot N(t)$$

A $^{22}$Na source has an activity of 1 μCi = $10^{-6}$ Ci,
how many $^{22}$Na nuclei are contained in the source?
(1 Ci = $3.7 \cdot 10^{10}$ decays/s)

$$N = \frac{A}{\lambda} = \frac{10^{-6}\,Ci}{8.5 \cdot 10^{-9}\ s^{-1}} = \frac{10^{-6} \cdot 3.7 \cdot 10^{10}\ s^{-1}}{8.5 \cdot 10^{-9}\ s^{-1}} = 4.36 \cdot 10^{12}$$

# How many grams of $^{22}$Na are in the source?

An amount of ***A*** grams of atoms with the mass number ***A*** (≅1mole) contains $N_A$ nuclei

$N_A$ ≡ Avogadro's Number = $6.023 \cdot 10^{23}$ nuclei/mole

⇨ 22g of $^{22}$Na contains $6.023 \cdot 10^{23}$ nuclei

$$N\left(^{22}Na\right) = 4.36 \cdot 10^{12} \ particles$$

$$1g = \frac{6.023 \cdot 10^{23}}{22} \ particles$$

$$N\left(^{22}Na\right) \equiv \frac{22 \cdot 4.36 \cdot 10^{12}}{6.023 \cdot 10^{23}} g = 1.59 \cdot 10^{-10} g$$

$$N(t) = N_0 \cdot e^{-\lambda \cdot t}$$

How many particles are in the source after 1 y, 2 y, 10 y?

$$N(t) = 4.36 \cdot 10^{12} \cdot e^{-0.27\,y^{-1} \cdot t}$$

$$A(t) = \lambda \cdot N(t) = 8.5 \cdot 10^{-9}\, s^{-1} \cdot N(t)$$

$$N(1y) = 4.36 \cdot 10^{12} \cdot e^{-0.27\,y^{-1} \cdot 1y} = 3.33 \cdot 10^{12}$$

$$A(1y) = 28305\, s^{-1} = 0.765 \mu Ci$$

$$N(2y) = 4.36 \cdot 10^{12} \cdot e^{-0.27\,y^{-1} \cdot 2y} = 2.54 \cdot 10^{12}$$

$$A(2y) = 21590\, s^{-1} = 0.58 \mu Ci$$

$$N(10y) = 4.36 \cdot 10^{12} \cdot e^{-0.27\,y^{-1} \cdot 10y} = 2.93 \cdot 10^{11}$$

$$A(10y) = 2490.5\, s^{-1} = 0.067 \mu Ci$$

Decay in particle number and corresponding activity!

# 2nd example: Radioactive Decay

Plutonium $^{239}$Pu, has a half life of 24,360 years.

1. What is the decay constant?
2. How much of 1kg $^{239}$Pu is left after t=100, 1,000, 10,000, 24,360, 100,000years?

$$\lambda = \frac{\ln 2}{t_{1/2}} = \frac{\ln 2}{24360\,y} = 2.85 \cdot 10^{-5}\,y^{-1}$$

$$N_{^{239}Pu}(t) = N_0 \cdot e^{-\lambda \cdot t} \Rightarrow N_{^{239}Pu}(100\,y) = 1kg \cdot e^{-2.85 \cdot 10^{-5}\,y^{-1} \cdot 100\,y}$$

$$N_{^{239}Pu}(100\,y) = 0.9972kg$$

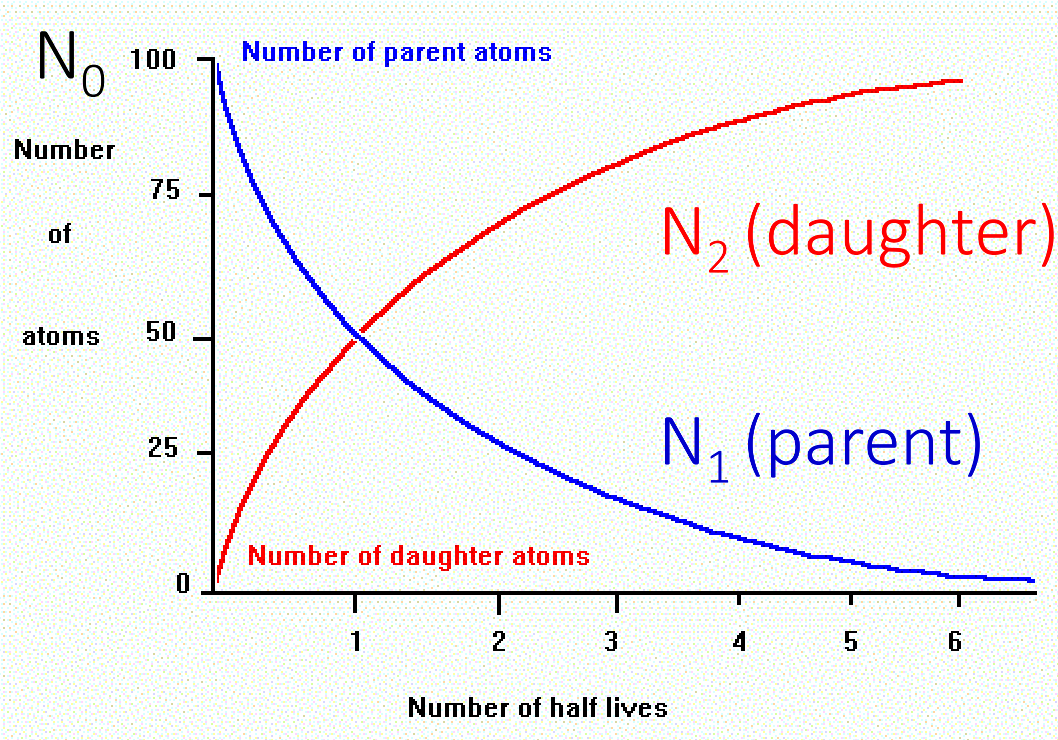
$$N_{^{239}Pu}(1{,}000\,y) = 0.9719kg$$

$$N_{^{239}Pu}(10{,}000\,y) = 0.7520kg$$

$$N_{^{239}Pu}(24{,}360\,y) = 0.5kg$$

$$N_{^{239}Pu}(100{,}000\,y) = 0.0578kg$$

# From parent to daughter nuclei




$^{14}C \Rightarrow {}^{14}N$

$T_{1/2}$=5,730 y

$^{22}Na \Rightarrow {}^{22}Ne$

$T_{1/2}$=2.6 y

$^{26}Al \Rightarrow {}^{26}Mg$

$T_{1/2}$=716,000 y

$^{40}K \Rightarrow {}^{40}Ar$

$T_{1/2}$=1,280,000,000 y

$$N_0 = N_1 + N_2, \quad N_2 = N_0 - N_1$$

$$N_1 = N_0 \cdot e^{-\lambda \cdot t}$$

$$N_2 = N_0 \cdot (1 - e^{-\lambda \cdot t})$$

The initial radioactive nuclei slowly decay with time converting the initial radioactive species to non radioactive material (or to yet another radioactive daughter nucleus).

# 3rd example: determine the number of daughter nuclei

Assume a mix of 100 nuclei of $^{14}C$, $^{22}Na$, $^{26}Al$, and $^{40}K$ each. Calculate the number of daughter nuclei after:
$t_1$=10 y, $t_2$=10,000 y, $t_3$=10,000,000 y and $t_4$=10,000,000,000 y

$$N_2 = N_0 \cdot (1 - e^{-\lambda \cdot t}) = N_0 \cdot (1 - e^{-\frac{\ln 2}{T_{1/2}} \cdot t})$$

| t | | | 10y | 10000 y | 10000000 y | 10000000000 y | |
|---|---|---|---|---|---|---|---|
| | $T_{1/2}$ | $\lambda$ | | | | | |
| $^{14}C$ | 5730 | 1.21E-04 | 1.21E-01 | 7.02E+01 | 1.00E+02 | 1.00E+02 | $^{14}N$ |
| $^{22}Na$ | 2.6 | 2.67E-01 | 9.30E+01 | 1.00E+02 | 1.00E+02 | 1.00E+02 | $^{22}Ne$ |
| $^{26}Al$ | 716000 | 9.68E-07 | 9.68E-04 | 9.63E-01 | 1.00E+02 | 1.00E+02 | $^{26}Mg$ |
| $^{40}K$ | 1280000000 | 5.42E-10 | 5.42E-07 | 5.42E-04 | 5.40E-01 | 9.95E+01 | $^{40}Ca/^{40}Ar$ |