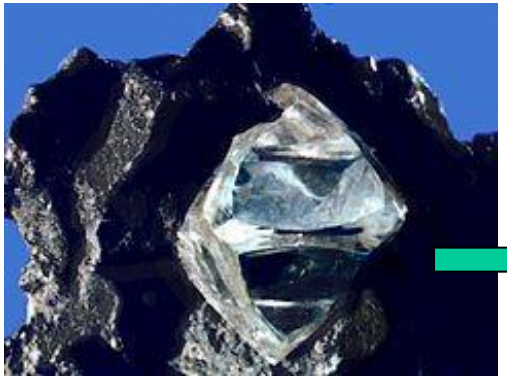
Radioactivity

Lecture 5

The Nature and Laws of Radioactivity

Changing Z to N or N to Z

Adding a proton (electron)





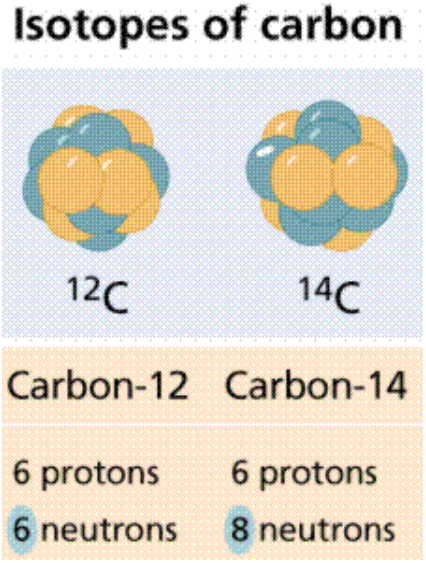
Carbon to Nitrogen



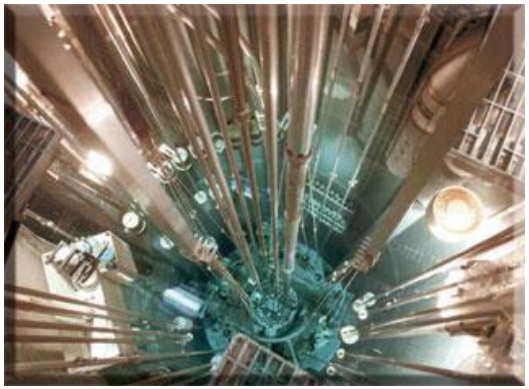
Gold to Mercury



Subtracting or adding neutrons



nucleus becomes unstable and decays by internally converting neutrons to protons (beta-decay)!





What are the physical laws that govern the decay process?

Terminology of nuclear decay

Time dependent change
from configuration 1 (radioactive nucleus)
To configuration 2 (decay product, daughter)



- Activity: number of decay events per time
- Decay constant: probability of decay
- Half life: time for the activity to be reduced to 50%

- Activity corresponds to the number of sand particles dripping through hole
- Decay constant is associated with the size of the hole



Units for the Activity A for a radioactive nucleus



- Classical unit: Curie: Ci corresponds to the number of decays of 1 g Radium as introduced by Madame Curie
- Modern unit: Becquerel: Bq notes a single decay event

$1\ \text{Ci} = 3.7 \cdot 10^{10}\ \text{decays/s} = 3.7 \cdot 10^{10}\ \text{Bq}$

$1\ \text{Bq} = 1\ \text{decay/s}$

Example: the human body is radioactive with and activity of:

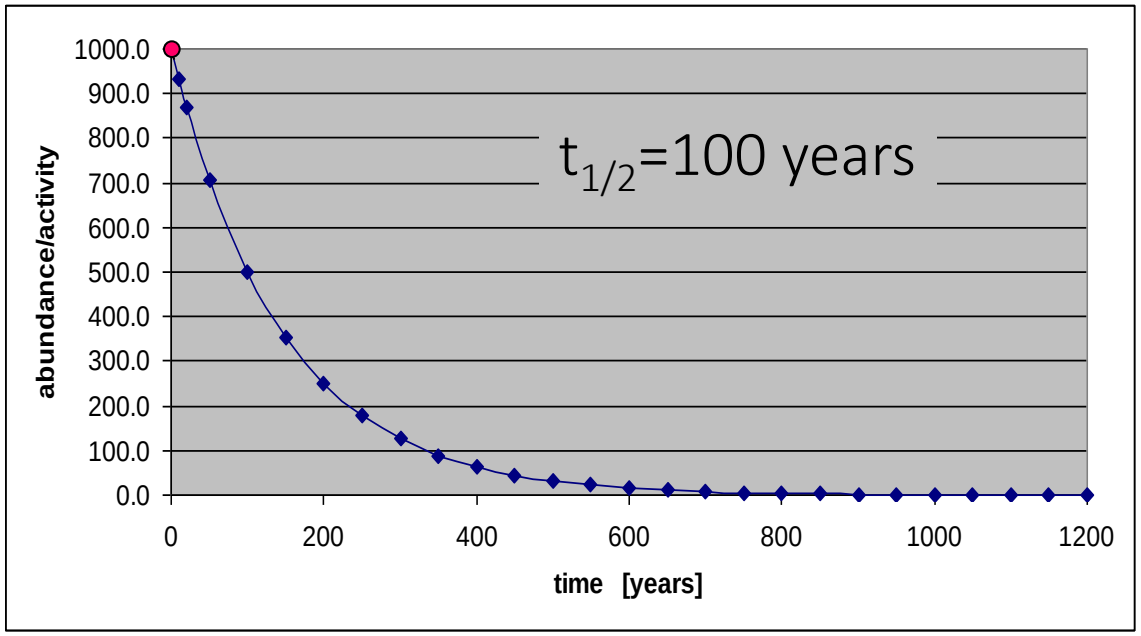
2.2·10-7Ci = 0.22μCi ⇒ 8000 Bq = 8 kBq

Sounds comfortably low sounds alarmingly high

Radioactive Decay Law



Describes the change of activity with time



$$A_{mother}(t) = A_0 \cdot e^{-\lambda \cdot t}$$

$$A_{daughter}(t) = A_0 \cdot \left(1 - e^{-\lambda \cdot t}\right)$$

exponential decay with time!
At half life 50% of the activity is gone!

λ≡decay constant;
a natural constant
for each radioactive
element.
Half life: $t_{1/2} = \ln 2/\lambda$

1st example: ^{22}Na

^{22}Na is a radioactive nucleus with a half-life of 2.6 years, what is the decay constant?
Mass number A=22; (don't confuse with activity *A(t)!)*

$$\lambda = \frac{\ln 2}{t_{1/2}} = \frac{\ln 2}{2.6\,y} = 0.27\,y^{-1}:$$

$$1\,y = 3.14 \cdot 10^{7}\,s \approx \pi \cdot 10^{7}\,s$$

$$\lambda = \frac{\ln 2}{2.6 \cdot 3.14 \cdot 10^{7}\,s} = 8.5 \cdot 10^{-9}\,s^{-1}$$

Radioactive Decay Laws

Activity of radioactive substance *A(t)* is at any time t proportional to number of radioactive particles *N(t)* :

$$A(t) = \lambda \cdot N(t)$$

A ^{22}Na source has an activity of 1 μCi = 10^{-6} Ci, how many ^{22}Na nuclei are contained in the source? (1 Ci = $3.7 \cdot 10^{10}$ decays/s)

$$N = \frac{A}{\lambda} = \frac{10^{-6} Ci}{8.5 \cdot 10^{-9}\, s^{-1}} = \frac{10^{-6} \cdot 3.7 \cdot 10^{10}\, s^{-1}}{8.5 \cdot 10^{-9}\, s^{-1}} = 4.36 \cdot 10^{12}$$

How many grams of ^{22}Na are in the source?

An amount of ***A*** grams of atoms with the mass number ***A*** (≅1mole) contains N_A nuclei

N_A ≡ Avogadro's Number = $6.023 \cdot 10^{23}$ nuclei/mole

⇨ 22g of ^{22}Na contains $6.023 \cdot 10^{23}$ nuclei

$$N\left(^{22}Na\right) = 4.36 \cdot 10^{12}\ particles$$

$$1g = \frac{6.023 \cdot 10^{23}}{22}\ particles$$

$$N\left(^{22}Na\right) \equiv \frac{22 \cdot 4.36 \cdot 10^{12}}{6.023 \cdot 10^{23}}\, g = 1.59 \cdot 10^{-10}\, g$$

$$N(t) = N_0 \cdot e^{-\lambda \cdot t}$$

How many particles are in the source after 1 y, 2 y, 10 y?

$N(t) = 4.36 \cdot 10^{12} \cdot e^{-0.27 y^{-1} \cdot t}$ $\quad A(t) = \lambda \cdot N(t) = 8.5 \cdot 10^{-9} s^{-1} \cdot N(t)$

$N(1y) = 4.36 \cdot 10^{12} \cdot e^{-0.27 y^{-1} \cdot 1y} = 3.33 \cdot 10^{12}$ $\quad A(1y) = 28305 s^{-1} = 0.765 \mu Ci$

$N(2y) = 4.36 \cdot 10^{12} \cdot e^{-0.27 y^{-1} \cdot 2y} = 2.54 \cdot 10^{12}$ $\quad A(2y) = 21590 s^{-1} = 0.58 \mu Ci$

$N(10y) = 4.36 \cdot 10^{12} \cdot e^{-0.27 y^{-1} \cdot 10y} = 2.93 \cdot 10^{11}$ $\quad A(10y) = 2490.5 s^{-1} = 0.067 \mu Ci$

Decay in particle number and corresponding activity!

2nd example: Radioactive Decay

Plutonium ^{239}Pu, has a half life of 24,360 years.

1. What is the decay constant?
2. How much of 1kg ^{239}Pu is left after t=100, 1,000, 10,000, 24,360, 100,000years?

$$\lambda = \frac{\ln 2}{t_{1/2}} = \frac{\ln 2}{24360y} = 2.85 \cdot 10^{-5}\, y^{-1}$$

$$N_{^{239}Pu}(t) = N_0 \cdot e^{-\lambda \cdot t} \Rightarrow N_{^{239}Pu}(100y) = 1kg \cdot e^{-2.85 \cdot 10^{-5}\, y^{-1} \cdot 100y}$$

$$N_{^{239}Pu}(100y) = 0.9972kg$$

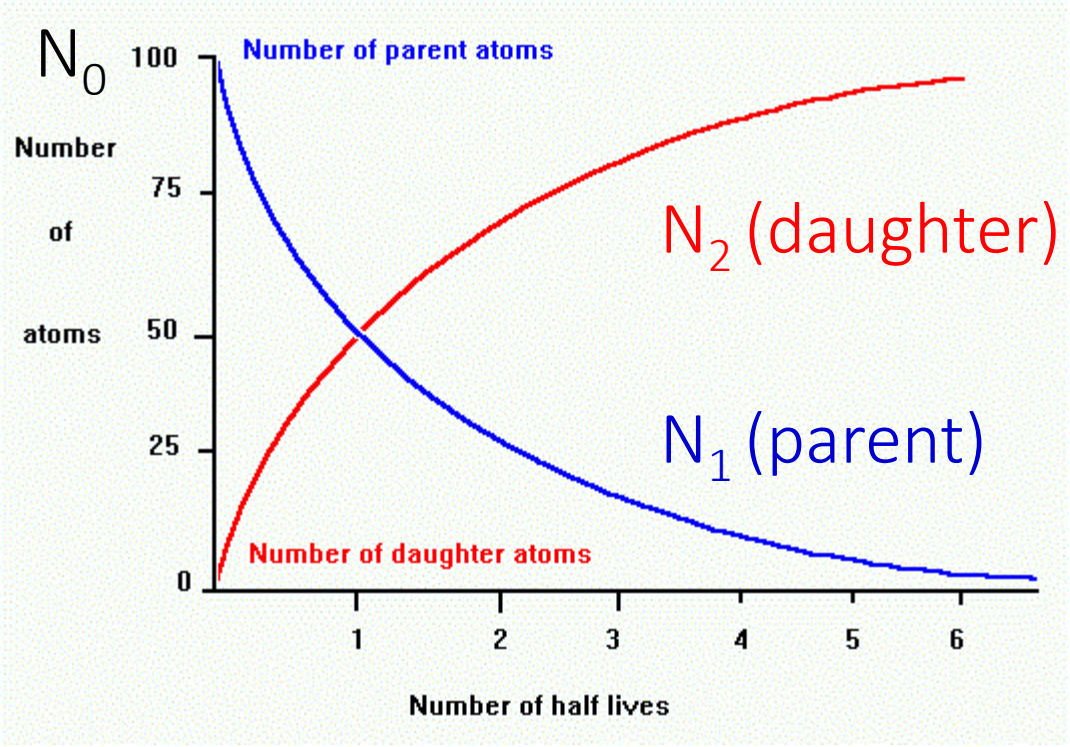
$$N_{^{239}Pu}(1{,}000y) = 0.9719kg$$

$$N_{^{239}Pu}(10{,}000y) = 0.7520kg$$

$$N_{^{239}Pu}(24{,}360y) = 0.5kg$$

$$N_{^{239}Pu}(100{,}000y) = 0.0578kg$$

From parent to daughter nuclei



$^{14}C \Rightarrow {}^{14}N$

$T_{1/2}$=5,730 y

$^{22}Na \Rightarrow {}^{22}Ne$

$T_{1/2}$=2.6 y

$^{26}Al \Rightarrow {}^{26}Mg$

$T_{1/2}$=716,000 y

$^{40}K \Rightarrow {}^{40}Ar$

$T_{1/2}$=1,280,000,000 y

$$N_0 = N_1 + N_2, \quad N_2 = N_0 - N_1$$

$$N_1 = N_0 \cdot e^{-\lambda \cdot t}$$

$$N_2 = N_0 \cdot (1 - e^{-\lambda \cdot t})$$

The initial radioactive nuclei slowly decay with time converting the initial radioactive species to non radioactive material

3rd example: determine the number of daughter nuclei

Assume a mix of 100 nuclei of ^{14}C, ^{22}Na, ^{26}Al, and ^{40}K each.
Calculate the number of daughter nuclei after:
t_1=10 y, t_2=10,000 y, t_3=10,000,000 y and t_4=10,000,000,000 y

$$N_2 = N_0 \cdot (1 - e^{-\lambda \cdot t}) = N_0 \cdot (1 - e^{-\frac{\ln 2}{T_{1/2}} \cdot t})$$

t			10y	10000 y	10000000 y	10000000000 y
	$T_{1/2}$	l				
^{14}N	5730	1.21E-04	1.21E-01	7.02E+01	1.00E+02	1.00E+02
^{22}Na	2.6	2.67E-01	9.30E+01	1.00E+02	1.00E+02	1.00E+02
^{26}Al	716000	9.68E-07	9.68E-04	9.63E-01	1.00E+02	1.00E+02
^{40}K	1280000000	5.42E-10	5.42E-07	5.42E-04	5.40E-01	9.95E+01