Radioactivity

Lecture 10
The radioactive Universe
Universe, Galaxy, Solar System, Earth
The Planck map of the Universe showing the temperature distribution in the background radiation from the Big Bang cooled down to an average temperature of 2.73 K. The small temperature fluctuations are in the 0.01% range and indicate slight inhomogeneities in the early universe.
The Cosmic Microwave Background decoupled from matter 400,000 years after the Big Bang. The Cosmic Neutrino Background decoupled after 1 second leaving relic neutrinos as direct signal from the nuclear particle processes in the first second. They have cooled now with the expansion of the universe to 1.95K which translates into an energy of about 100-200μeV. All other neutrinos from stellar nuclear reaction and decay processes have higher energies! The relic neutrino flux (number of neutrinos penetrating every square centimeter of our body per second) is up to 20 trillion/cm²⋅sec. About 10 million share the volume of your and my bodies at any time. But there is no harm, the probability for damage is exceedingly small.
Our Radioactive Galaxy

Infrared radiation range

Distribution of heat sources (stars) along the galactic plane

Gamma radiation range

Distribution of radioactive sources along the galactic plane
$^{26}$Al and $^{60}$Fe as long-lived isotopes

$^{26}$Al is a neutron deficient Aluminum isotope ($^{27}$Al is stable) with a half-life of 716,000 years. It is produced by proton induced reactions on stable $^{25}$Mg isotopes during the last phases of stellar burning in massive stars and during the shock-front expansion of supernovae. $^{60}$Fe has a lifetime of $2.6 \cdot 10^6$ years and is produced by neutron capture on the stable $^{58}$Fe isotope. The decay lines of both isotopes are observed by satellite based gamma detectors like INTEGRAL and RHESSI.

The estimated steady state production rates are $2.0 \pm 1.0 \text{ M}_\odot$/Myr for $^{26}$Al and $0.75 \pm 0.4 \text{ M}_\odot$/Myr for $^{60}$Fe. This corresponds to $2.2 \pm 1.1 \text{ M}_\odot$ of $^{26}$Al and $1.7 \pm 0.9 \text{ M}_\odot$ of $^{60}$Fe in the present interstellar medium. Predictions for the $^{60}$Fe mass distribution, total mass, and flux map are given, in particular a $^{60}$Fe/$^{26}$Al flux ratio of $0.16 \pm 0.12$. 
The galactic activity from $^{26}$Al and $^{60}$Fe

The solar mass $M_\odot = 1.989 \cdot 10^{30}$ kg

$M(^{26}\text{Al}) = 2.2 \cdot M_\odot = 2.2 \cdot 1.989 \cdot 10^{30}$ kg $= 4.38 \cdot 10^{30}$ kg

$N(^{26}\text{Al}) = \frac{N_A}{A} \cdot M(^{26}\text{Al}) = \frac{6.022 \cdot 10^{23}}{26 \cdot 10^{-3} \text{ kg}} \cdot M(^{26}\text{Al}) = 10^{56}$ nuclei

$A(^{26}\text{Al}) = \lambda \cdot N(^{26}\text{Al}) = \frac{\ln 2}{T_{1/2}} \cdot N(^{26}\text{Al}) = \frac{0.69}{7.16 \cdot 10^5 \cdot 3.14 \cdot 10^7 \text{ s}} \cdot 10^{56} = 3.11 \cdot 10^{42}$ Bq

$M(^{60}\text{Fe}) = 1.7 \cdot M_\odot = 1.7 \cdot 1.989 \cdot 10^{30}$ kg $= 3.38 \cdot 10^{30}$ kg

$N(^{60}\text{Fe}) = \frac{N_A}{A} \cdot M(^{60}\text{Fe}) = \frac{6.022 \cdot 10^{23}}{60 \cdot 10^{-3} \text{ kg}} \cdot M(^{60}\text{Fe}) = 3.4 \cdot 10^{55}$ nuclei

$A(^{60}\text{Fe}) = \lambda \cdot N(^{60}\text{Fe}) = \frac{\ln 2}{T_{1/2}} \cdot N(^{60}\text{Fe}) = \frac{0.69}{2.6 \cdot 10^6 \cdot 3.14 \cdot 10^7 \text{ s}} \cdot 3.4 \cdot 10^{55} = 2.87 \cdot 10^{41}$ Bq
Radioactive decay patterns of $^{26}$Al, $^{44}$Ti, and $^{60}$Fe

Emission of the 1.8 MeV $\gamma$ ray from the ground-state transition in the daughter nucleus $^{26}$Mg.

Emission of the 1.17 MeV and the 1.33 MeV $\gamma$ lines from the subsequent $^{60}$Co decay to $^{60}$Ni.

Emission of the 1.157 MeV $\gamma$ line from the subsequent $^{44}$Sc decay to $^{44}$Ca.
The radioactivity of $^{44}$Ti in Cassiopeia A

A core collapse supernova such as Cassiopeia A (Cas A) that exploded 336 years ago ejects up to $10^{-4} \cdot M_\odot$ on $^{44}$Ti into interstellar space. The ejected $^{44}$Ti can easily be calculated:

$$M(^{44}Ti) = 10^{-4} \cdot M_\odot = 10^{-4} \cdot 1.989 \cdot 10^{30} \text{ kg} = 2 \cdot 10^{26} \text{ kg}$$

$$N(^{44}Ti) = \frac{N_A}{A} \cdot M(^{44}Ti) = \frac{6.022 \cdot 10^{23}}{44 \cdot 10^{-3} \text{ kg}} \cdot M(^{44}Ti) = 2.7 \cdot 10^{51} \text{ particles}$$

$$A(^{44}Ti) = \lambda \cdot N(^{44}Ti) = \frac{\ln 2}{T_{1/2}} \cdot N(^{44}Ti) = \frac{0.69}{59 \cdot 3.14 \cdot 10^7 \text{ s}} \cdot 10^{56} = 1.01 \cdot 10^{42} \text{ Bq}$$

What is the $^{44}$Ti activity emitting $\gamma$ radiation today?

$$A(^{44}Ti)_{now} = A(^{44}Ti)_{1680 AD} \cdot e^{-\lambda \cdot t} = 1.01 \cdot 10^{42} \text{ Bq} \cdot e^{\frac{-0.69}{59 \cdot 336 \text{ y}}} = 2 \cdot 10^{40} \text{ Bq}$$
Supernova light curve
Radioactive $^{56}$Ni in core collapse supernovae

Core collapse supernovae produce up to $1 \, M_{\odot}$ on $^{56}$Ni, depending on the mass of the progenitor star. The half-life of $^{56}$Ni is 6.1 days, it therefore decays faster than the examples discussed before, but releases its energy at a much faster time scale which explains the SN light curve.

\[
M^{(56}\text{Ni}) = 1 \cdot M_{\odot} = 1.989 \cdot 10^{30} \text{ kg} = 2 \cdot 10^{30} \text{ kg}
\]

\[
N^{(56}\text{Ni}) = \frac{N_A}{A} \cdot M^{(56}\text{Ni}) = \frac{6.022 \cdot 10^{23}}{56 \cdot 10^{-3} \text{ kg}} \cdot M^{(56}\text{Ni}) = 2.15 \cdot 10^{55} \text{ particles}
\]

\[
A^{(56}\text{Ni}) = \lambda \cdot N^{(56}\text{Ni}) = \frac{\ln 2}{T_{1/2}} \cdot N^{(56}\text{Ni}) = \frac{0.69}{6.1 \cdot 8.64 \cdot 10^4 \text{ s}} \cdot 2.15 \cdot 10^{55} = 2.81 \cdot 10^{49} \text{ Bq}
\]

Today, 29 years (or 10 thousand days) later the $^{56}$Ni activity is gone, replaced by the activity of $^{56}$Co ($T_{1/2}=77.3 d$) and superseded by the activity of the long-lived $^{44}$Ti ($T_{1/2}=59 y$).

\[
A^{(56}\text{Ni})_{\text{now}} = A^{(56}\text{Ni})_{1987AD} \cdot e^{-\lambda \cdot t} = 2.81 \cdot 10^{49} \text{ Bq} \cdot e^{-\frac{0.69}{6.1d} \cdot 10502d} = 0 \text{ Bq}
\]

\[
A^{(56}\text{Co})_{\text{now}} = A^{(56}\text{Co})_{1987AD} \cdot e^{-\lambda \cdot t} = 2.81 \cdot 10^{49} \text{ Bq} \cdot e^{-\frac{0.69}{77.3d} \cdot 10502d} = 5.45 \cdot 10^8 \text{ Bq}
\]
Decay law for longer lived daughter nuclei

\[ N_2(t) = N_0 \cdot \frac{\lambda_1}{\lambda_2 - \lambda_1} \left( e^{-\lambda_1 t} - e^{-\lambda_2 t} \right) \]

with \( \lambda_1 = \frac{\ln 2}{T_{1/2(1)}} \) \( \quad \lambda_2 = \frac{\ln 2}{T_{1/2(2)}} \)

Daughter nucleus \(^{56}\text{Co}\) is formed by the decay of the \(^{56}\text{Ni}\) mother nucleus and maintains the overall level of activity or energy release depending on its lifetime.

My supernova is hundred times more powerful than SN1887a, because of the initial guess of 1 M\(_\odot\) on \(^{56}\text{Ni}\), with a \(^{56}\text{Ni}\) production of 0.01 M\(_\odot\) good agreement is obtained!
Energy production powering the light curve

The energy $Q$ is produced by the radioactive decay of the $^{56}\text{Ni}$ to $^{56}\text{Fe}$, which is facilitated by electron capture as alternative to $\beta^+$ decay; the energy release for each decay process corresponds to the mass difference.

Each decay by electron capture generates energy that corresponds to the mass difference between $^{56}\text{Ni}$ and the final decay product $^{56}\text{Fe}$.

$$Q = m^{(56}\text{Ni}) \cdot c^2 - m^{(56}\text{Fe}) \cdot c^2 = (55.942u - 55.935u) \cdot c^2 \quad 1u = 1.66054 \cdot 10^{-27} \text{ kg}$$

$$c^2 = 8.99 \cdot 10^{16} \frac{m^2}{s^2} \quad Q = 1.05 \cdot 10^{-12} J = 6.55 \text{ MeV}$$

with: $1u = 931.502 \text{ MeV} / c^2 \quad Q = 6.55 \text{ MeV}$

3.8 MeV emitted as $\gamma$ radiation

That is the energy released by a single decay event, what is the total energy released in a supernovae?

Energy units: $1 \text{ kg} \frac{m^2}{s^2} = 1 \text{ J} = 6.24 \cdot 10^{12} \text{ MeV} = 10^7 \text{ erg}$

$$Q_{total} = Q \cdot A(t) \quad A(t_0) = 2.81 \cdot 10^{49} \text{ Bq}$$

$$Q_{total} = 1.84 \cdot 10^{50} \frac{\text{MeV}}{s} = 2.95 \cdot 10^{37} \frac{J}{s} = 2.95 \cdot 10^{44} \frac{\text{erg}}{s}$$
Uranium Thorium Radioactivities

From model predictions and observations supernova produces only a small fraction of very heavy elements above Fe. Average values are:

$$\frac{Pb}{Fe} = 3.2 \cdot 10^{-4} \quad \frac{Th}{Fe} = 2 \cdot 10^{-5} \quad \frac{U}{Fe} = 5 \cdot 10^{-6}$$

with

$$N(Fe) \approx 10^{56} \text{ particles}$$

$$N(Th) \approx 2 \cdot 10^{51} \text{ particles}$$

$$N(U) \approx 5 \cdot 10^{50} \text{ particles}$$

$$T_{1/2}^{(232Th)} = 1.4 \cdot 10^{10} \text{ y}$$

$$T_{1/2}^{(238U)} = 4.5 \cdot 10^{9} \text{ y}$$

$$A^{(232Th)} = \lambda^{(232Th)} \cdot 2 \cdot 10^{51} = \frac{\ln 2}{1.4 \cdot 10^{10} \text{ y} \cdot 3.14 \cdot 10^{7}} \cdot 2 \cdot 10^{51}$$

$$A^{(232Th)} = 3.14 \cdot 10^{33} \text{ Bq}$$

$$A^{(238U)} = \lambda^{(238U)} \cdot 5 \cdot 10^{50} = \frac{\ln 2}{4.5 \cdot 10^{9} \text{ y} \cdot 3.14 \cdot 10^{7}} \cdot 5 \cdot 10^{50}$$

$$A^{(238U)} = 2.44 \cdot 10^{33} \text{ Bq}$$

Comparable activities because of balance between abundance and half-life but only about a billionth of the activity from $^{26}Al$ and $^{60}Fe$ observed in our universe!