Beam Optics & RIB Separators

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Exotic Beam Summer School
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Why beams are useful

Nuclear Physics is an experimental science. We often need particle beams to study **many** nuclei in our experiments.

- ➔ **Measurement uncertainties**
  Imperfect measurement tools will bring uncertainties.
- ➔ **Quantum mechanical reality**
  We measure quantities that have intrinsic widths.
- ➔ **Experiment through interaction**
  Structure and dynamics are often revealed through interactions with other nuclei (collisions at finite energy).

What is a beam?

An experimental conveyor belt.

A group of particles with a “small” phase space.

- Occupy a small space (in x and y)
- Moving in a similar direction (similar momentum vectors)

Small phase space

Think of physical situations where a group of things might satisfy the requirements to be called a “beam.”

Which of these examples could be thought of as a “beam,” occupying a small phase space?

1. The molecules of gas in this room
2. A swarm of bees coming in and out of their hive
3. Cars and trucks moving along the interstate highway
4. Photons emitted by a laser pointer
5. Alpha particles scattering from gold nuclei in a foil

The language of beam dynamics

Assume a direction of travel: the "optic axis"

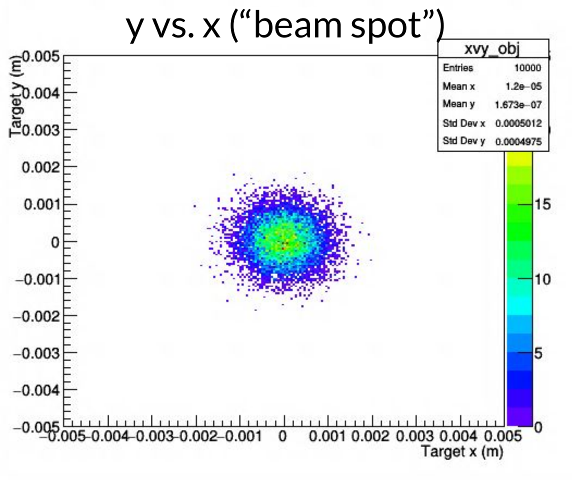
ALL coordinates are measured from this.

- Transverse positions
  - x - horizontal
  - y - vertical
- Transverse angles of travel
  - a - horizontal deflection (or θ)
  - b - vertical deflection (or φ)

Small phase space!

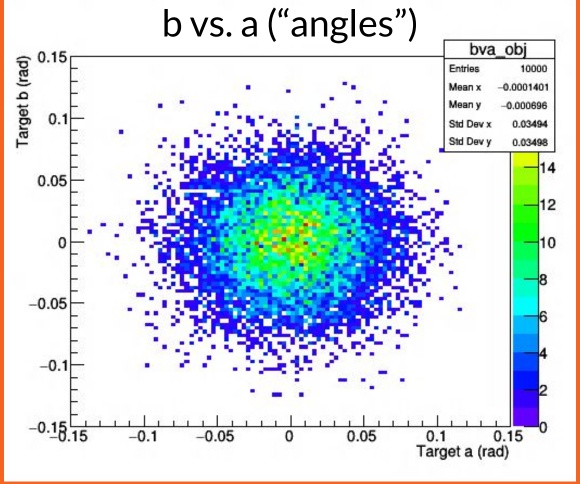
Again, we assume that these coordinates are **small** for a beam of particles (i.e. particles are all **near** the optic axis and move **nearly** along it).

Examples (simulated) of 2D projections of beam phase spaces.



What you might see on a viewer plate in the beamline.

Would need tracking detectors to see this.



The language of beam dynamics

Other particle properties relative to the optic axis and the central ray

- Central ray is along the optic axis with: $K = K_o$, $q = q_o$
- Kinetic energy or momentum deviation: δ_K or δ_p
  - Relative to K_o, the kinetic energy of the central ray
- Time-of-flight deviation: Δt
  - Relative to t_o, the time-of-flight of the central ray

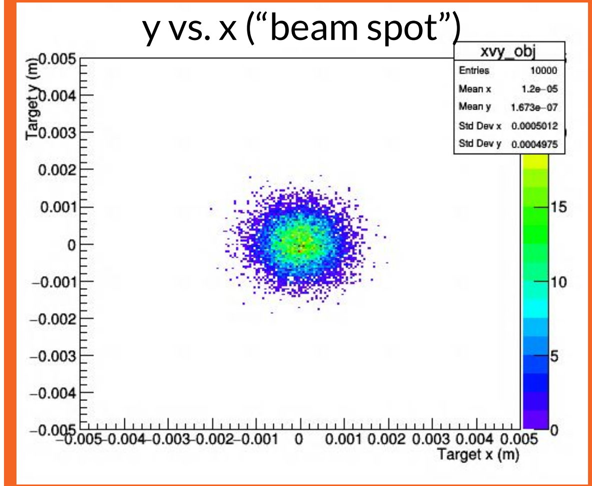
$$\delta_K = (K - K_o)/K_o$$

$$\Delta t = t - t_o$$

Is it possible for a particle to be on the optic axis but not on the central ray?

1. Yes
2. No
3. It depends on whether we’re in the upside down.

Examples (simulated) of 2D projections of beam phase spaces.



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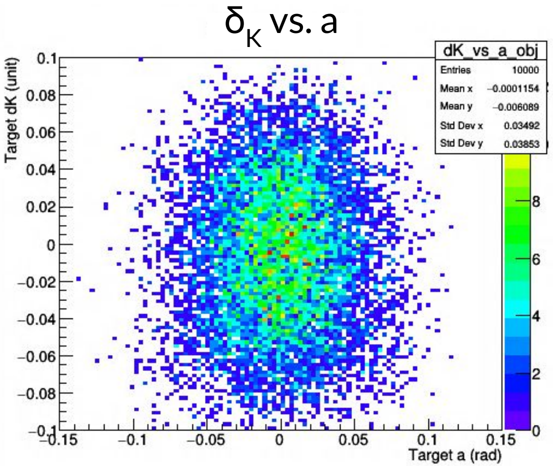
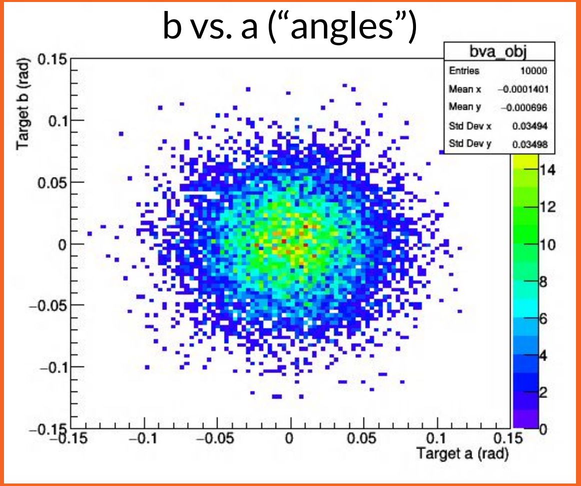


Energy vs. angle distribution

Would need tracking detectors to see this.



How do we act on beam particles?

Magnetic and electric fields exert a force (the Lorentz force) on **charged particles**.

➔ Magnetic force

$$\vec{F} = q\vec{v} \times \vec{B}$$

➔ Electric force

$$\vec{F} = q\vec{E}$$

➔ Notable features

Magnetic force scales with the velocity.

Both forces scale with the charge of the beam particle.

Controlling the beam with E and B

- Higher charge state (*q*) makes both E- and B-fields more effective at bending trajectories
- Higher energy beams -> use magnetic fields
  - Typical B $\lesssim$ 2T for iron-yoke magnets
  - Much higher fields possible for air-core magnets
- Lower energy beams -> use electric fields
  - E-fields limited by electrostatic breakdown
  - Simpler construction for low fields

$$\vec{F} = q\vec{v} \times \vec{B}$$

$$\vec{F} = q\vec{E}$$

E and B -> Particle trajectories

- **Magnetic rigidity:** $B\rho = \dfrac{p}{q}$
- **Electric rigidity:** $E\rho = \dfrac{2K}{q}$

Thus we can get the bend radius due to an applied field on a particle.

B - Magnetic field strength

ρ - radius of curvature of beam trajectory

p - momentum

q - charge

E - Electric field strength

K - Kinetic energy

Given the two relations below about the effect of a magnetic field on a particle trajectory, what can we say about the <u>bend radius vs. particle velocity</u> for a constant B and q?

$$\vec{F}_{\mathrm{B}} = q\vec{v} \times \vec{B} \qquad\qquad B\rho = \frac{p}{q}$$

1. Particles of higher velocity will be bent more sharply (smaller ρ).
2. Particles of higher velocity will be bent less sharply (larger ρ).
3. Particles of all velocities will be bent with the same ρ.
4. These relations do not lead to a clear answer.

What field shapes do we use in beam optics?

Magnetic and electric multipoles would be a general way to refer to the field configurations. The most popular types:

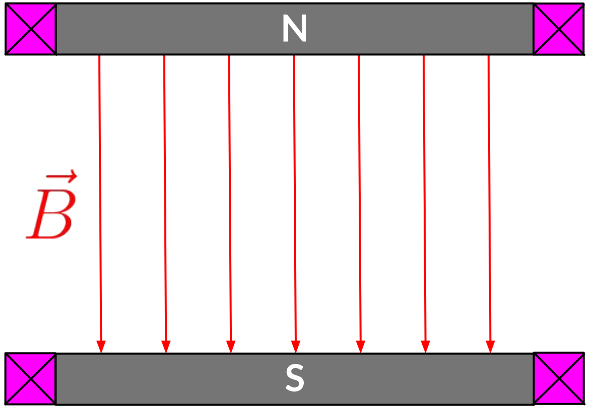
➔ **Dipoles**
for bending and separating

➔ **Quadrupoles**
for focusing and defocusing
[stop here for first order optics]

➔ **Higher order multipoles**
for addressing aberrations
sextupoles, octupoles, etc.

Magnetic Dipole fields (Uniform B-fields)

Typically vertical fields
Producing horizontal bends



Which way would a *positive* beam bend heading into the screen?

Beam view of idealized dipole B-field

- **Dipoles bend the optic axis of the system**
- **Dipoles separate or "disperse" particles by rigidity:**

Magnetic dipoles separate p/q (or "momentum")

Electric dipoles separate K/q (or "kinetic energy")

Fun fact:

Combining Electric and Magnetic fields lets you **separate by mass**, a **very useful** thing indeed.

Magnetic quadrupole fields
(magnetic lenses)

S N

midplane symmetry

N S

Let's draw a quadrupole field!

- **Midplane symmetry**

  Orienting magnetic poles such that the fields have a symmetry axis on the horizontal plane with:

  $$B_x(y) = -B_x(y)$$

  $$B_y(y) = B_y(y)$$

Magnetic quadrupole fields
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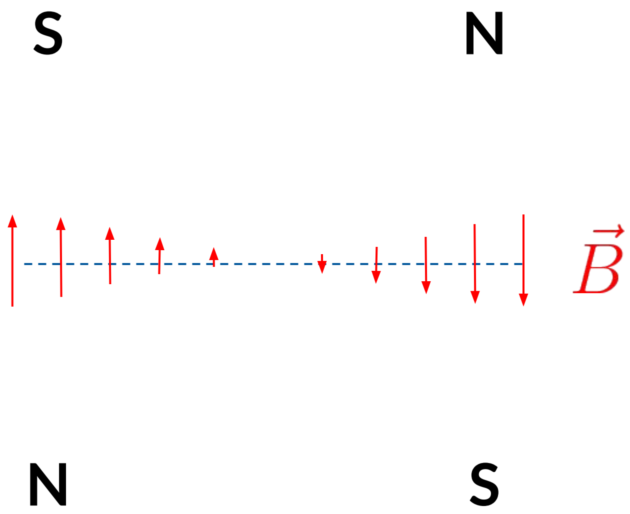

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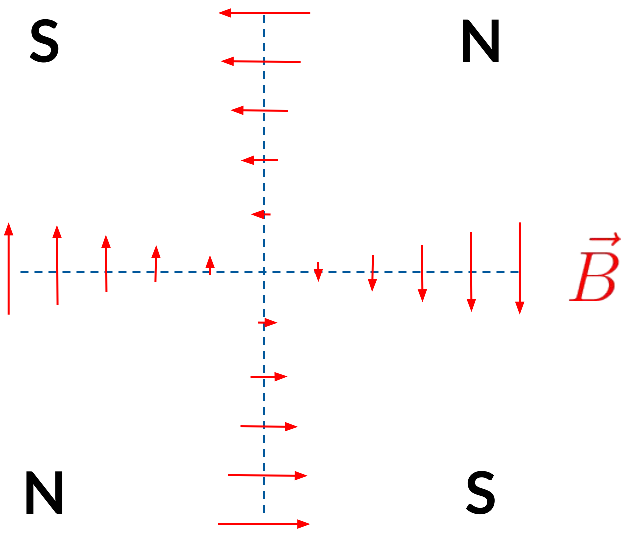
- **Quadrupole field strength depends linearly on distance from the center**

Magnetic quadrupole fields
(magnetic lenses)



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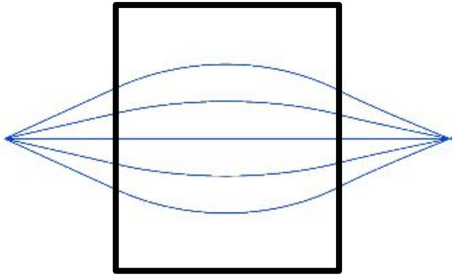
- **Quadrupole field strength depends linearly on distance from the center**

  Quadrupoles that **focus** horizontally will **de-focus** vertically (and vice versa)

Assume a quadrupole vertically focuses a beam at a given distance from its exit. How would the trajectories and the focal length change for particles of higher momentum, *p*?



$$B\rho = \frac{p}{q}$$

1. They will be bent more sharply and focused later.
2. They will be bent more sharply and focused sooner.
3. They will be bent less sharply and focused later.
4. They will be bent less sharply and focused sooner.

How do we describe the action of an optical system on a beam?

Taylor expansions are applied, since the multi-dimensional phase space is contained within small deviations from the central ray.

➔ First order Taylor coefficients

Give a first-order transfer map

Describe basic optics properties

Where is the focal plane?

What is the dispersion in *p*?

➔ Higher order coefficients

Important for large emittances

[RIB typically have large emittances]

Taylor Expansion

$$\vec{r} = \begin{pmatrix} x_i \\ a_i \\ y_i \\ b_i \\ t_i \\ \delta_K \end{pmatrix}$$

$$x_f = (x|x)x_i + (x|a)a_i + (x|y)y_i + (x|b)b_i + (x|\delta_K)\delta_K$$

Initial position

[single particle]

Final position

Linearly dependent on each initial coordinate

Taylor Expansion

$$\vec{r} = \begin{pmatrix} x_i \\ a_i \\ y_i \\ b_i \\ t_i \\ \delta_K \end{pmatrix}$$

$$x_f = (x|x)x_i + \boxed{(x|a)}a_i + (x|y)y_i + (x|b)b_i + (x|\delta_K)\delta_K$$

Coefficients depend on the beam optical system design and field settings.

$$(x|a) \equiv (x_f|a_i)$$

Initial position

[single particle]

Final position

Linearly dependent on each initial coordinate

Taylor Expansion

$$\vec{r} = \begin{pmatrix} x_i \\ a_i \\ y_i \\ b_i \\ t_i \\ \delta_K \end{pmatrix}$$

$$\begin{aligned}
x_f &= (x|x)x_i + (x|a)a_i + (x|y)y_i + (x|b)b_i + (x|\delta_K)\delta_K \\
a_f &= (a|x)x_i + (a|a)a_i + (a|y)y_i + (a|b)b_i + (a|\delta_K)\delta_K \\
y_f &= (y|x)x_i + (y|a)a_i + (y|y)y_i + (y|b)b_i + (y|\delta_K)\delta_K \\
b_f &= (b|x)x_i + (b|a)a_i + (b|y)y_i + (b|b)b_i + (b|\delta_K)\delta_K \\
t_f &= (t|x)x_i + (t|a)a_i + (t|y)y_i + (t|b)b_i + (t|\delta_K)\delta_K
\end{aligned}$$

Initial position

[single particle]

Final position

Linearly dependent on each initial coordinate

Taylor Expansion

$$\vec{r} = \begin{pmatrix} x_i \\ a_i \\ y_i \\ b_i \\ t_i \\ \delta_K \end{pmatrix}$$

$$\begin{aligned}
x_f &= (x|x)x_i + (x|a)a_i + \cancel{(x|y)y_i + (x|b)b_i} + (x|\delta_K)\delta_K \\
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y_f &= \cancel{(y|x)x_i + (y|a)a_i} + (y|y)y_i + (y|b)b_i + \cancel{(y|\delta_K)\delta_K} \\
b_f &= \cancel{(b|x)x_i + (b|a)a_i} + (b|y)y_i + (b|b)b_i + \cancel{(b|\delta_K)\delta_K} \\
t_f &= (t|x)x_i + (t|a)a_i + \cancel{(t|y)y_i + (t|b)b_i} + (t|\delta_K)\delta_K
\end{aligned}$$

Initial position

[single particle]

Final position

Linearly dependent on each initial coordinate

Many terms are zero due to midplane symmetry

Taylor Expansion

$$\vec{r}_f = \begin{pmatrix} x_f \\ a_f \\ y_f \\ b_f \\ t_f \\ \delta_K \end{pmatrix} = \begin{pmatrix} (x|x) & (x|a) & (x|y) & (x|b) & (x|t) & (x|\delta_K) \\ (a|x) & (a|a) & (a|y) & (a|b) & (a|t) & (a|\delta_K) \\ (y|x) & (y|a) & (y|y) & (y|b) & (y|t) & (y|\delta_K) \\ (b|x) & (b|a) & (b|y) & (b|b) & (b|t) & (b|\delta_K) \\ (t|x) & (t|a) & (t|y) & (t|b) & (t|t) & (t|\delta_K) \\ (\delta_K|x) & (\delta_K|a) & (\delta_K|y) & (\delta_K|b) & (\delta_K|t) & (\delta_K|\delta_K) \end{pmatrix} \begin{pmatrix} x_i \\ a_i \\ y_i \\ b_i \\ t_i \\ \delta_K \end{pmatrix}$$

Final position
[single particle]

Transfer Map
[beam optical system]

Initial position
[single particle]

Taylor Expansion

$$\vec{r}_f = \begin{pmatrix} x_f \\ a_f \\ y_f \\ b_f \\ t_f \\ \delta_K \end{pmatrix} = \begin{pmatrix} (x|x) & (x|a) & 0 & 0 & 0 & (x|\delta_K) \\ (a|x) & (a|a) & 0 & 0 & 0 & (a|\delta_K) \\ 0 & 0 & (y|y) & (y|b) & 0 & 0 \\ 0 & 0 & (b|y) & (b|b) & 0 & 0 \\ (t|x) & (t|a) & 0 & 0 & (t|t) & (t|\delta_K) \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x_i \\ a_i \\ y_i \\ b_i \\ t_i \\ \delta_K \end{pmatrix}$$

Final position
[single particle]

Transfer Map
[beam optical system]

Initial position
[single particle]

Taylor Expansion

$$\vec{r}_f = \begin{pmatrix} x_f \\ a_f \\ y_f \\ b_f \\ t_f \\ \delta_K \end{pmatrix} = \left(\begin{array}{cc|cc|cc} (x|x) & (x|a) & 0 & 0 & 0 & (x|\delta_K) \\ (a|x) & (a|a) & 0 & 0 & 0 & (a|\delta_K) \\ \hline 0 & 0 & (y|y) & (y|b) & 0 & 0 \\ 0 & 0 & (b|y) & (b|b) & 0 & 0 \\ \hline (t|x) & (t|a) & 0 & 0 & (t|t) & (t|\delta_K) \\ 0 & 0 & 0 & 0 & 0 & 1 \end{array}\right) \begin{pmatrix} x_i \\ a_i \\ y_i \\ b_i \\ t_i \\ \delta_K \end{pmatrix}$$

Final position [single particle] | **Transfer Map [beam optical system]** | **Initial position [single particle]**

Notice: the horizontal and vertical motion are completely independent in first order, given mid-plane symmetry.

Important Transfer Map Terms and Values

- **$(x|a) = 0$**

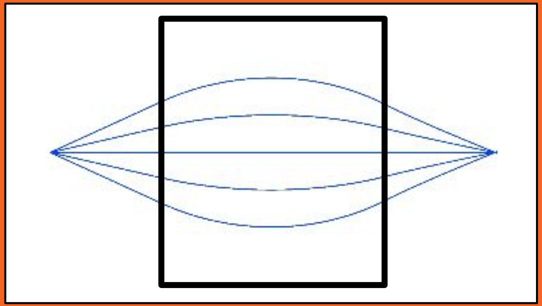
  point-to-point focus in horizontal
- **$(?|?)$ = 0** **What map element?**

  point-to-point focus in vertical
- **$(x|x) = M_x$ and $(y|y) = M_y$**

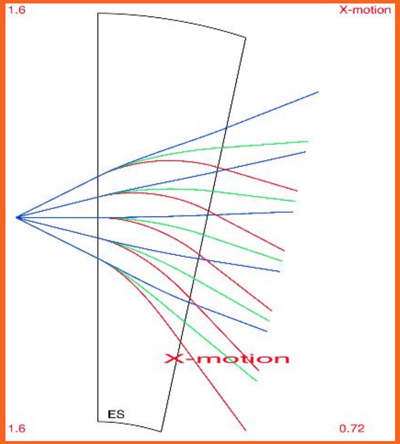
  magnifications (horiz. and vert.)
- **$(a|a)$ and $(b|b)$**

  angular magnifications (H and V)
- **$(x|\delta_K)$ "dispersion" - physical**

  separation by particle energy



An Image



Dispersion

Important Transfer Map Terms and Values

- **$(x|a) = 0$**

  point-to-point focus in horizontal

- **$(y|b) = 0$**

  point-to-point focus in vertical

- **$(x|x) = M_x$ and $(y|y) = M_y$**

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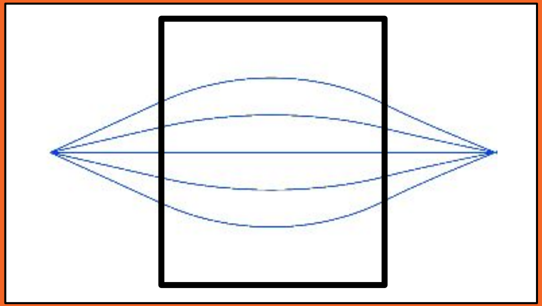

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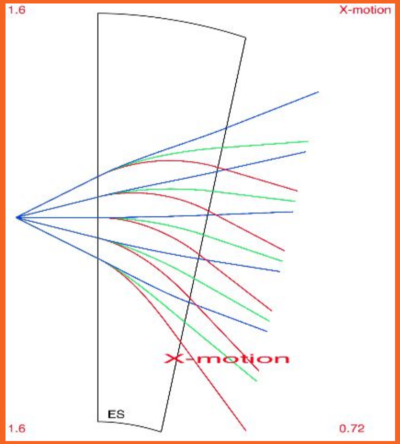


Dispersion

We want separation at an image

- $(x|a) = 0$

  point-to-point focus in horizontal
- $(y|b) = 0$

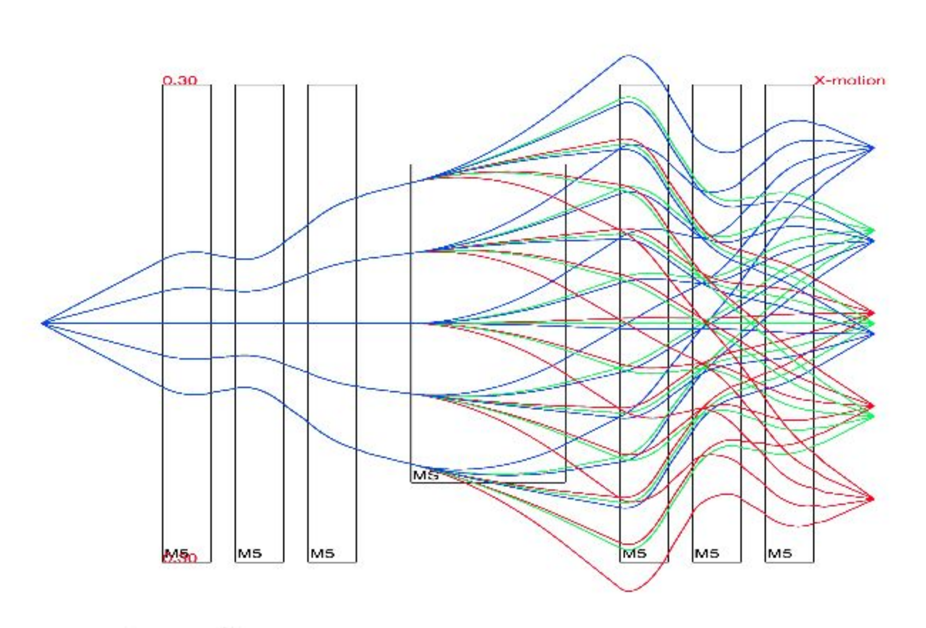
  point-to-point focus in vertical
- $(x|x) = M_x$ and $(y|y) = M_y$

  magnifications (horiz. and vert.)
- $(a|a)$ and $(b|b)$

  angular magnifications (H and V)
- $(x|\delta_K)$ “dispersion” - physical

  separation by particle energy

Resolving Power



Different colors correspond to different particle energies

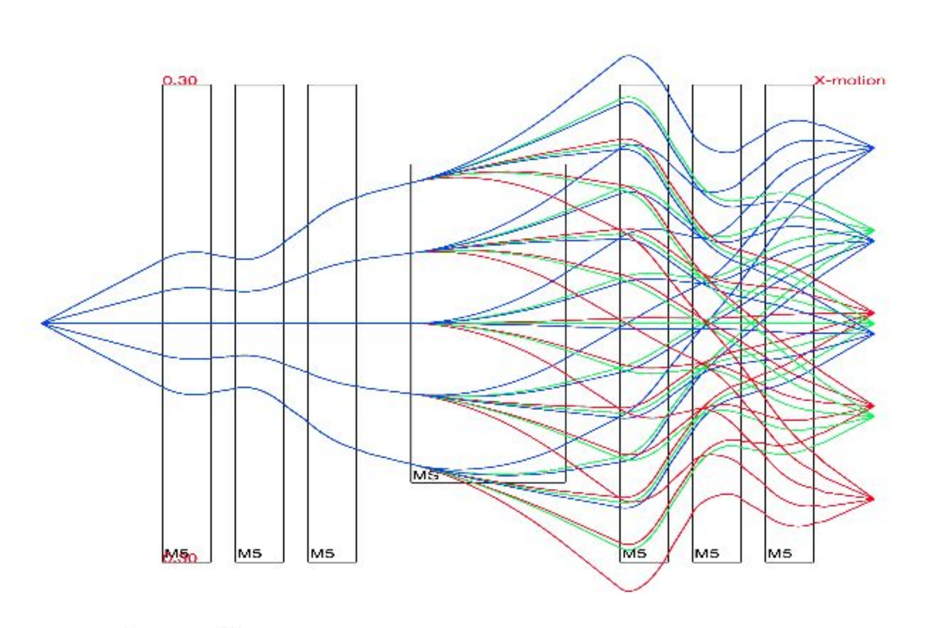
Resolving power

dispersion

$$R = \frac{(x|\delta_K)}{2\sigma_{xo}(x|x)}$$

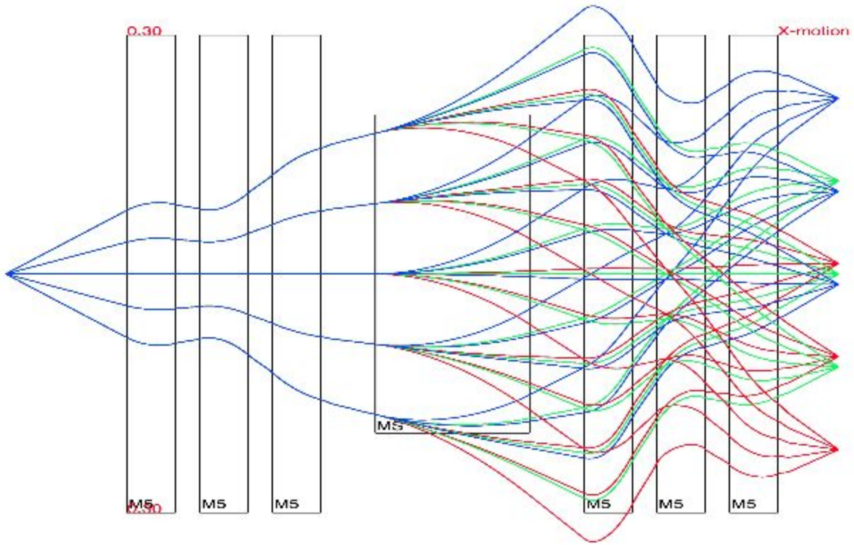
Initial spot width

magnification



Different colors correspond to different particle energies

If the dispersion (separation) induced by the dipole in the system is only in energy (δ_K), why are there three separate spots for each energy?



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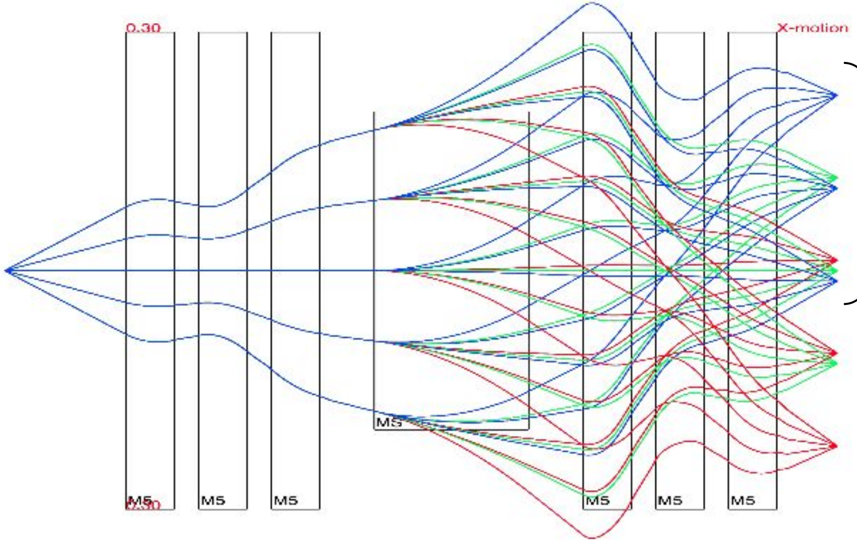


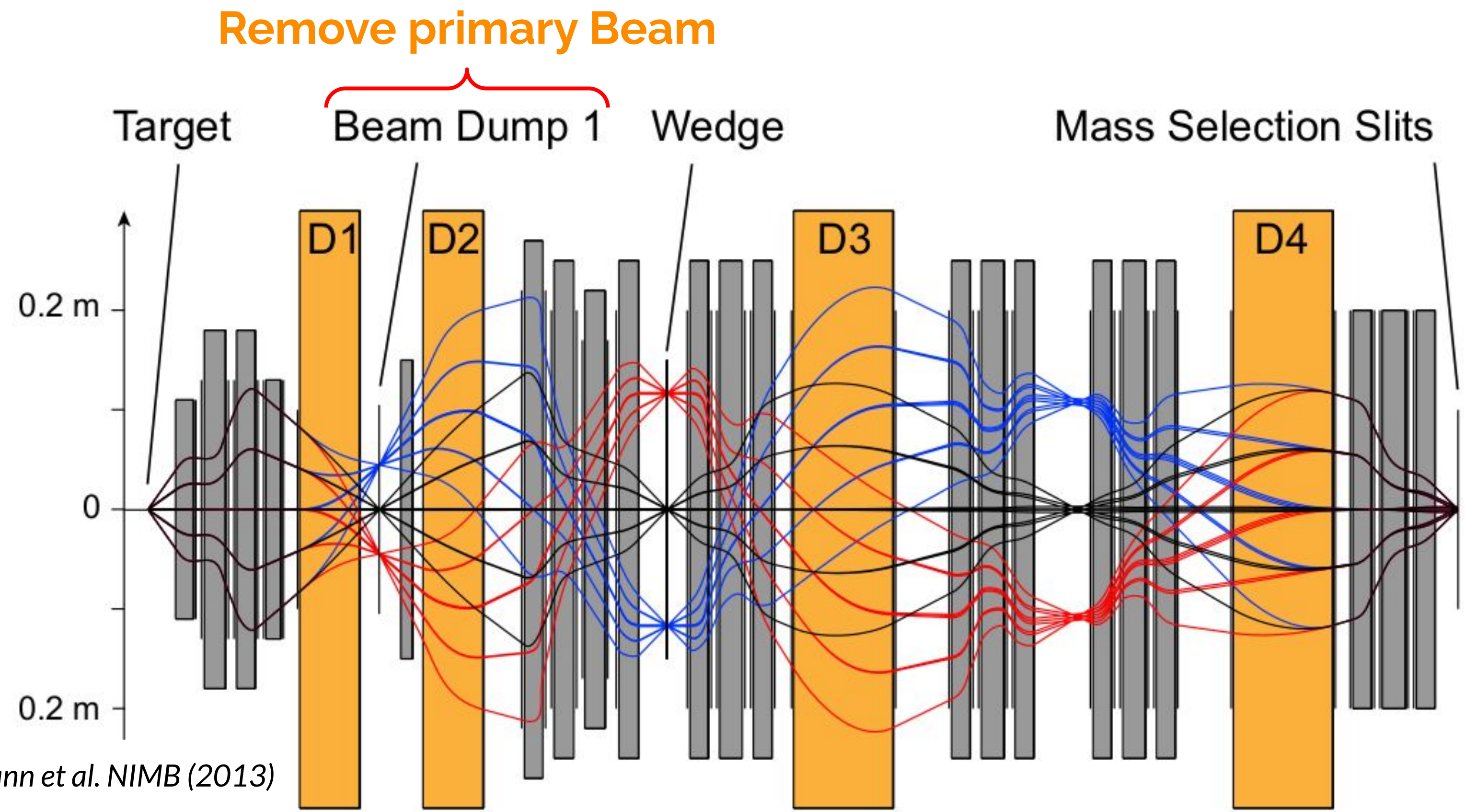
3 distinct *q* values (charge states) for each δ_K value shown

$$B\rho = \frac{p}{\boxed{q}}$$

What do we want an RIB separator to do?

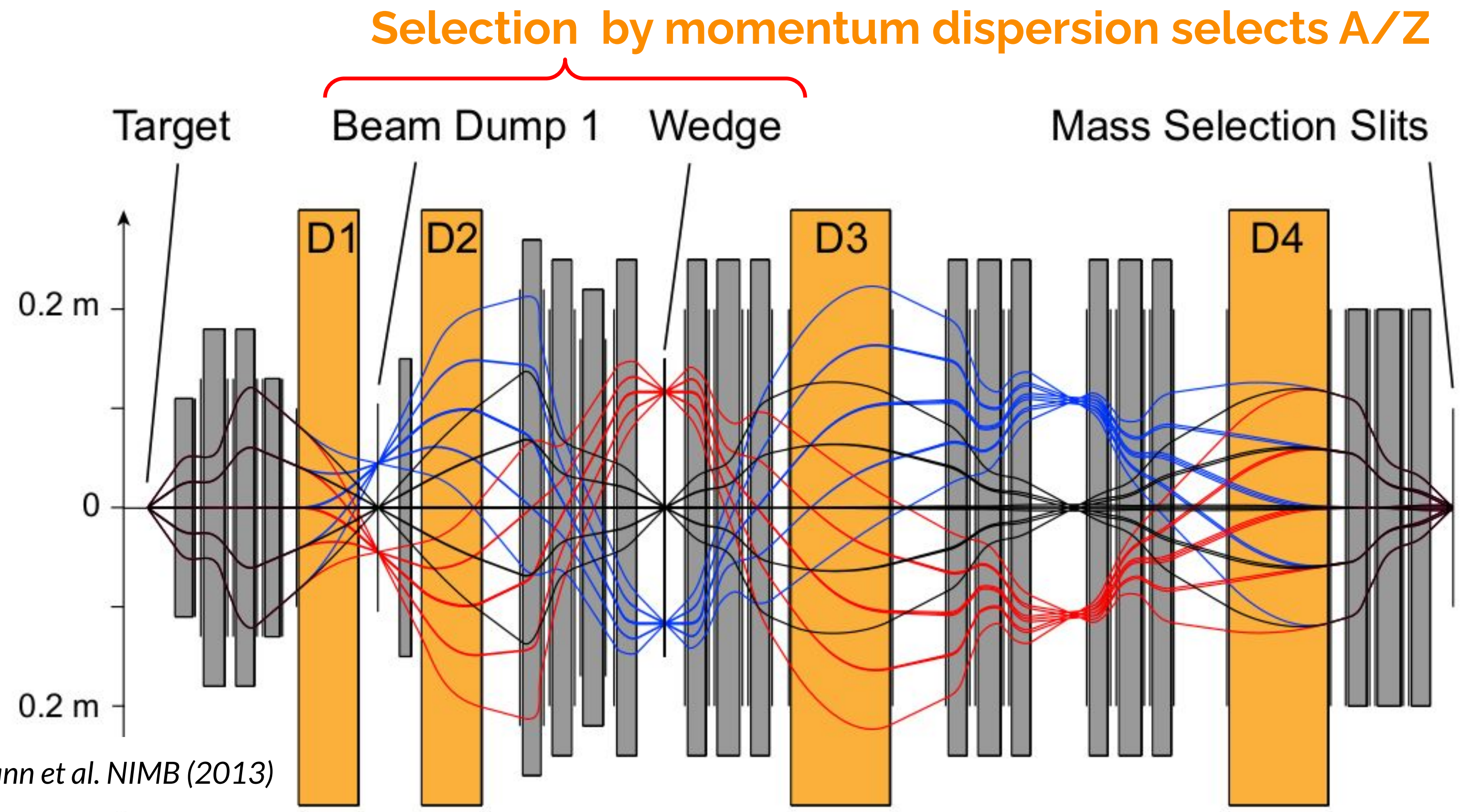
- ➔ **Reject the primary beam**
  So it doesn't destroy things
- ➔ **Accept and focus the products**
  Large enough apertures
  And careful tuning
- ➔ **Separate reaction products**
  Often physically, rejecting some
- ➔ **Measure particle properties**
  Z, A, q, K, p, v, m, a, b
  Individual particle-by-particle measurements can be used and correlated with other detectors

Separator Example - ARIS at FRIB



M. Hausmann et al. NIMB (2013)

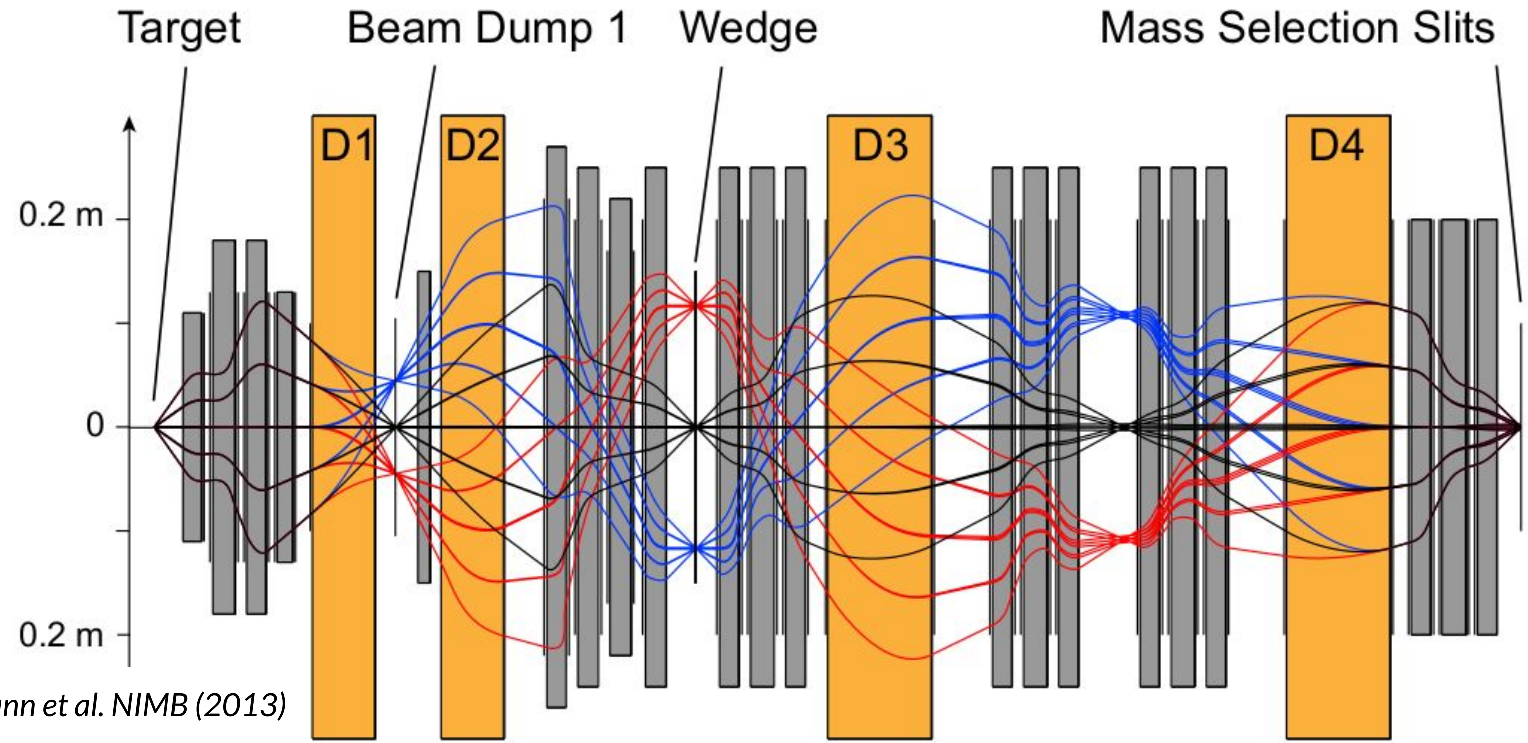
Separator Example - ARIS at FRIB



M. Hausmann et al. NIMB (2013)

Separator Example - ARIS at FRIB

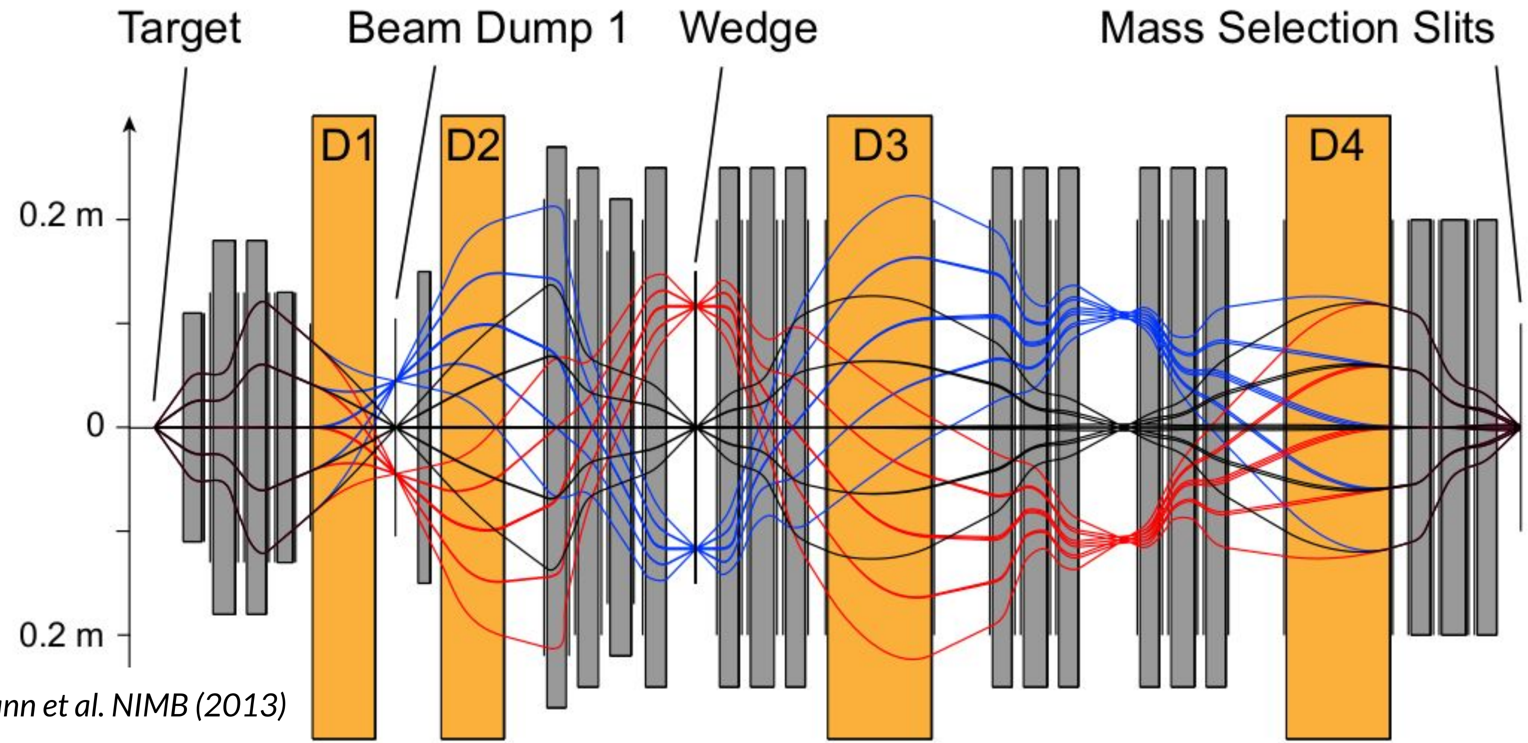
Momentum loss in “wedge” degrader depends on Z



M. Hausmann et al. NIMB (2013)

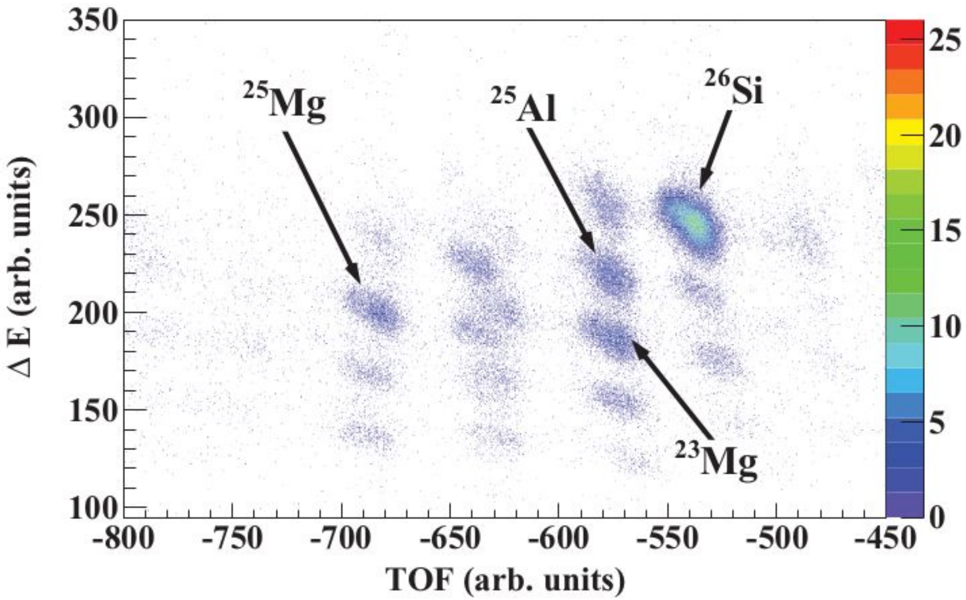
Separator Example - ARIS at FRIB

Finally transmit a small region of A and Z



M. Hausmann et al. NIMB (2013)

Measuring particle properties



J. Chen et al. PRC 85, 045809 (2012)

Particles can be further sorted by particle-by-particle tracking:

- Energy loss
- Time-of-flight
- Velocity
- TKE
- Momentum
- Angles
- Position

Rare Isotope Beam vs. Stable Beam

- Multiple Species: A, Z, Q (~100)
- Large energy spread
- Large angular spread
- Complicated time structure*
- Higher order optics are important (need sext. & oct.)

More types of larger aperture magnets in longer systems (for multi-stage separation)

- Single Species*
- Small energy spread
- Small angular spread
- Well-defined time structure*
- First order optics often sufficient

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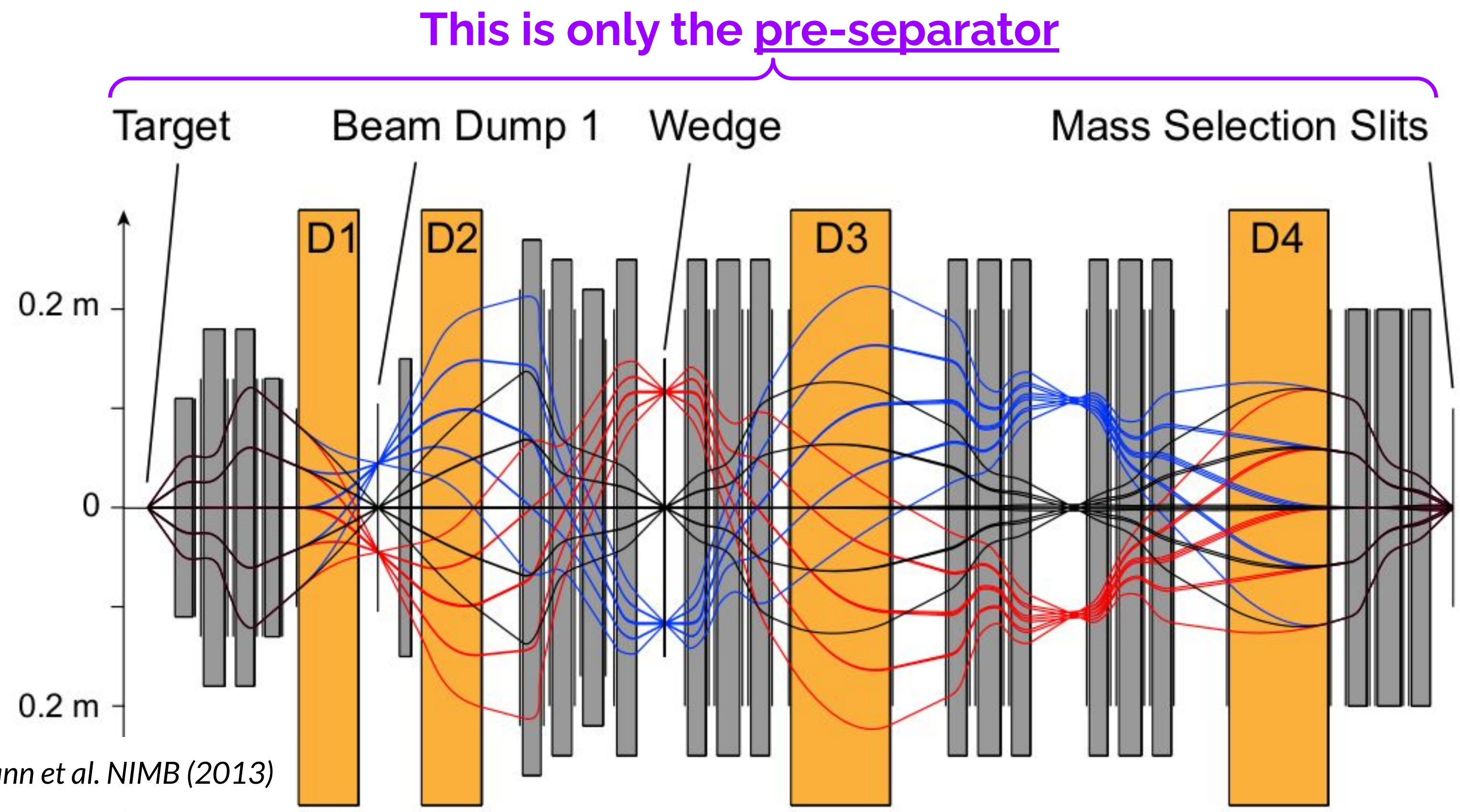
*often, not always

Separator Example - ARIS at FRIB



M. Hausmann et al. NIMB (2013)

You can't always get what you want.

Magnetic and electric multipoles would be a general way to refer to the field configurations. The most popular types:

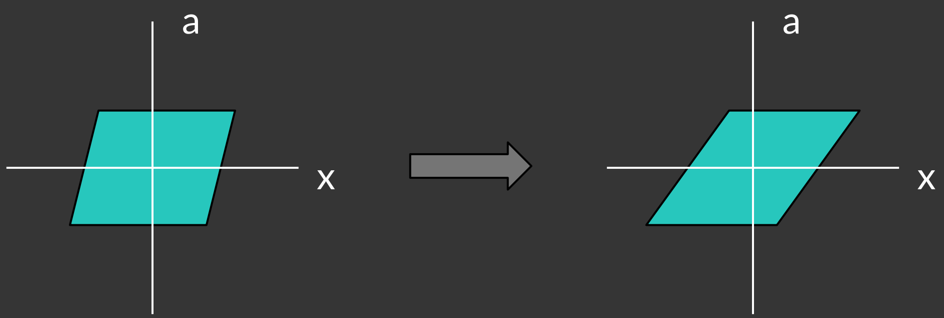
- ➔ Liouville's theorem
  You can't shrink the phase space
- ➔ Field quality
  A dipole isn't always just a dipole
- ➔ Apertures and field strengths
  Cost, field quality, and resolving power req. all limit acceptance

System limitation - Liouville's Theorem

The volume of phase space occupied by the particle beam is invariant when traversing electromagnetic fields.*

$$\iint dx_f da_f = \iint dx_i da_i$$

$$\iint dy_f db_f = \iint dy_i db_i$$

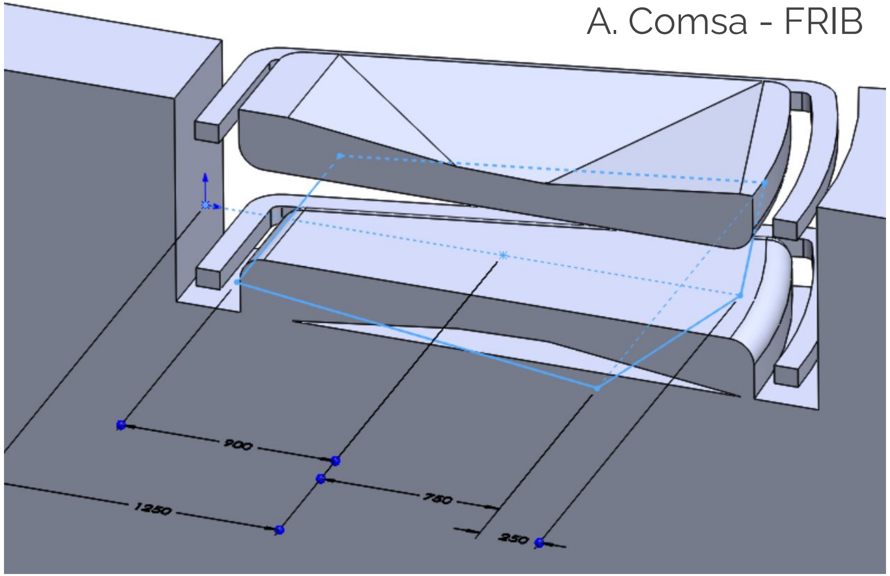


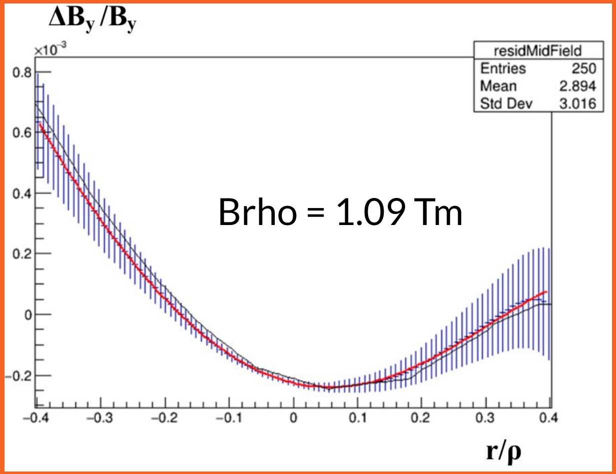
*In systems with no energy gain or loss (i.e. no acceleration and no degrader materials)

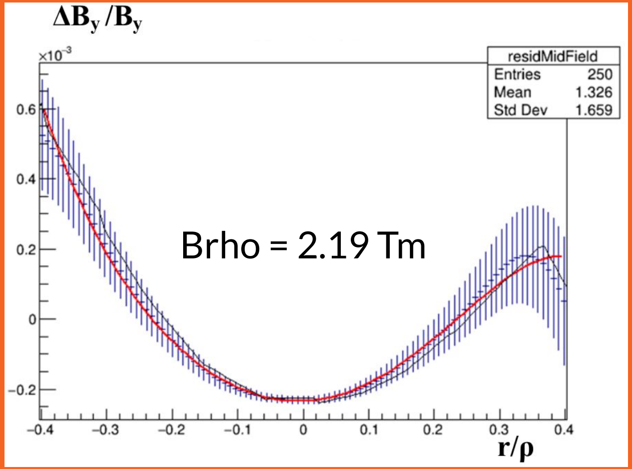
Field Quality

➔ **Field inside the dipole isn't uniform**
- Curved lines show residual field.
- These act like higher order multipoles.
- The shape will be field dependent.

A. Comsa - FRIB



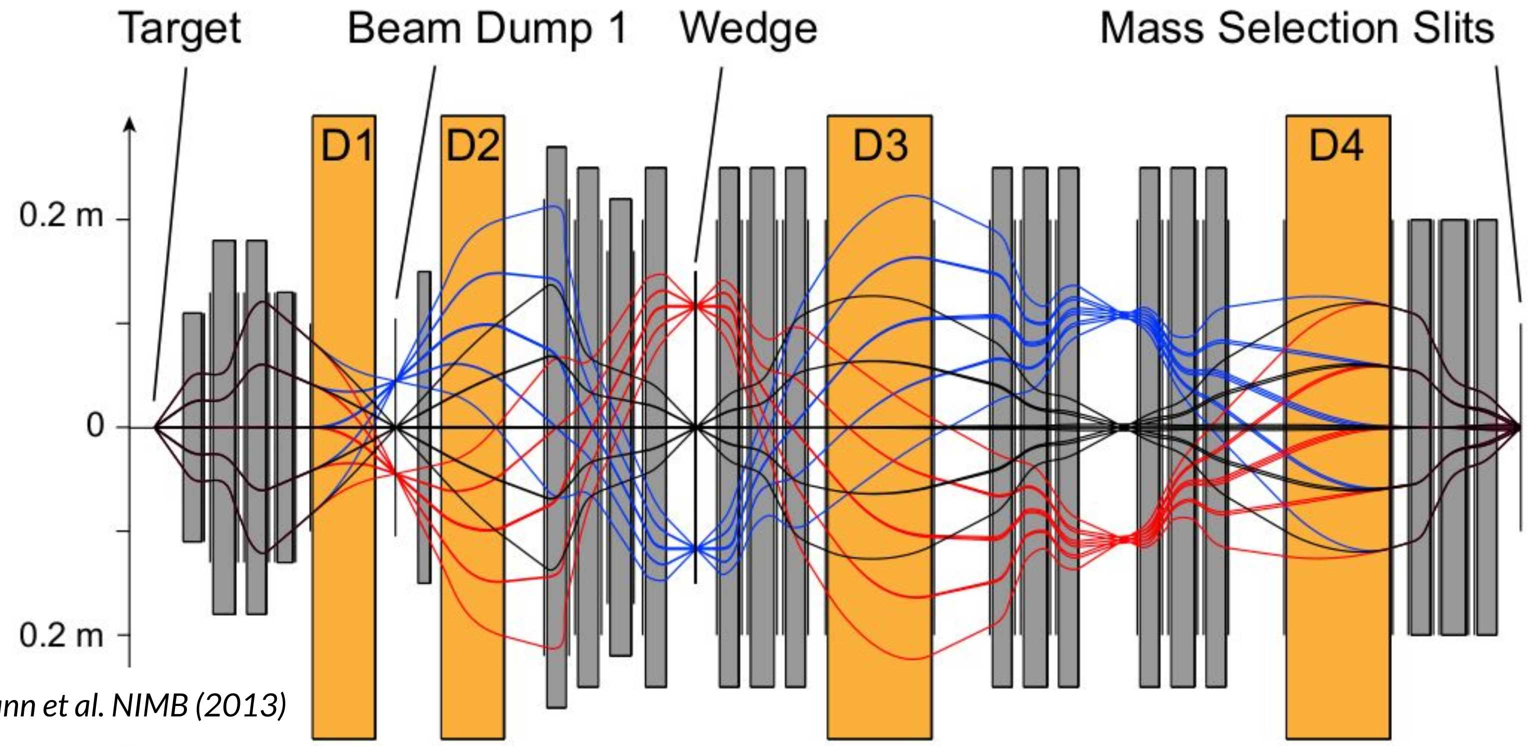




– Acceptance limits - from element apertures

Element apertures -> limit <u>angular acceptance</u> (σ_a, σ_b)

Apertures where dispersed -> limit <u>momentum acceptance</u> (δ_p, δ_K)



M. Hausmann et al. NIMB (2013)

What we've seen:

Beam optics allows us to transmit, separate, and identify the products of nuclear reactions:

- ➔ Using E and B fields
  Dipole, quadrupole, sextupole…
- ➔ Taylor expansion description
  Framework to define beam optical system properties
- ➔ Within limits of theory/systems
  Liouville's theorem, acceptance, field quality, resolving power

More questions?