Nuclear structure theory I: Foundations and phenomena

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UNIVERSITY OF NOTRE DAME

Central challenge: Starting from quantum chromodynamics (QCD), can we derive the properties of nuclei (and matter)?

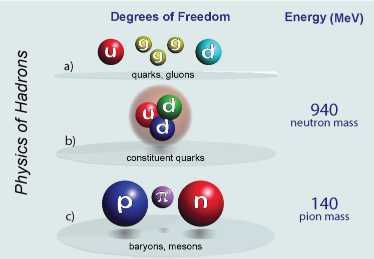
NUCLEAR PHYSICS

Low energy: Nuclear structure, nuclear reactions & astrophysics

Intermediate energy: Nucleon and hadronic structure

Relativistic heavy ion collisions & quark matter

Neutrino physics, the standard model, and beyond



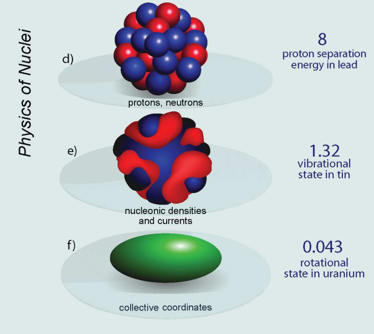
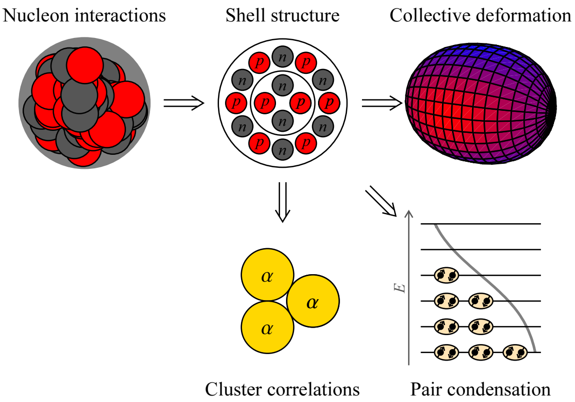


Figure from *The Frontiers of Nuclear Science: A Long-Range Plan*, NSF/DOE NSAC (2007).



Many-particle Schrödinger equation

$$\sum_{i=1}^{A}\left(-\frac{\hbar^2}{2m_i}\nabla_i^2\right)\Psi + \frac{1}{2}\sum_{i,j=1}^{A} V(|\mathbf{r}_i-\mathbf{r}_j|)\,\Psi = E\Psi$$

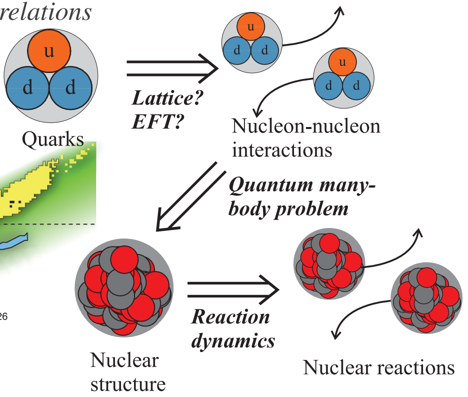
$$\Psi(\mathbf{r}_1,\mathbf{r}_2,\mathbf{r}_3,\ldots,\mathbf{r}_A) \;=\; ?$$

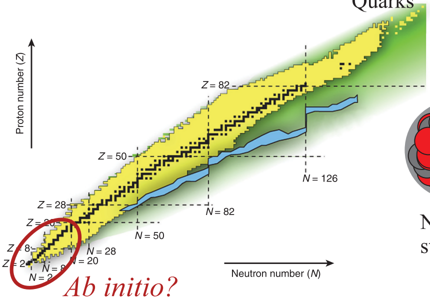
Goal of *ab initio* nuclear structure

First-principles understanding of nature *Nuclei from QCD*

Can we understand the origin of "simple patterns in complex nuclei"?

i.e., emergent collective correlations





Adapted from B. Schwarzschild, Physics Today 63(8), 16 (2010).

Some comments on nuclear structure

The nucleus is fundamentally a quantum many-body system determined by its constituents (nucleons) and their interactions

We are forced to subject this many-body problem to brutal approximations

Robust, simple patterns emerge, in form of collective correlations

Symmetries and symmetry breaking frequently provide an organizing principle *"Physics is symmetries"*

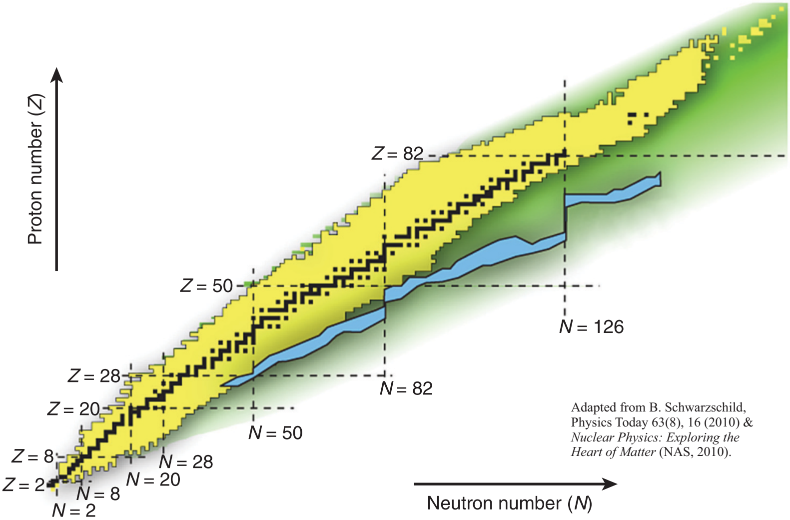
Structure bridges energy scales for first-principles understanding of nature (*ab initio*?)

Structure underlies reactions (astrophysics, applications) and interactions (electroweak, beyond the standard model)

Nuclear structure is part of quantum many-body theory (study of condensates)

Outline

- Nuclear structure tour
- Shell model as baseline framework for structure
- Observables and illustrations



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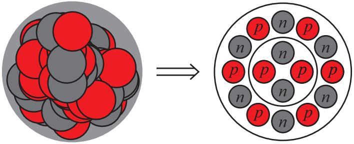
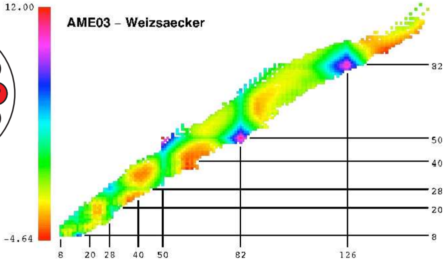
"Magic" numbers: 2, 8, 20, 28, 50, 82, 126

From simple shell structure to collective dynamics





P. Van Isacker, AIP Conf. Proc. No. 819 (2006), p. 57.

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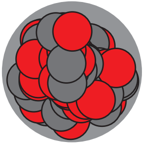

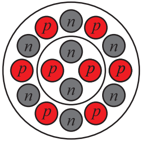

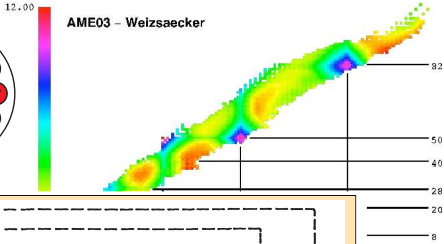

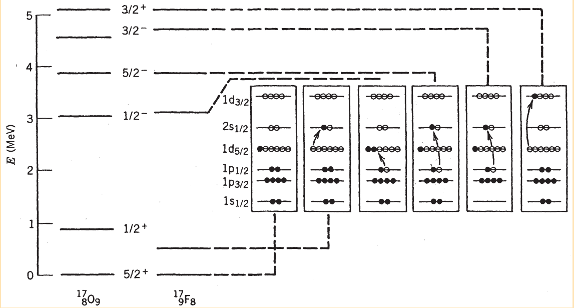

Kenneth S. Krane, *Introductory Nuclear Physics* (Wiley, New York, 1988).

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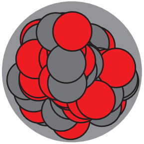

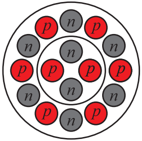

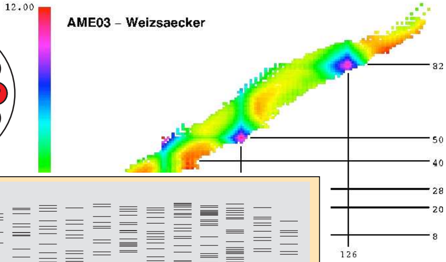

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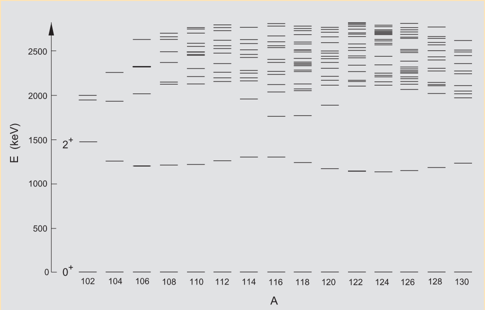

David J. Rowe and John L. Wood, *Fundamentals of Nuclear Models* (World Scientific, Singapore, 2010).

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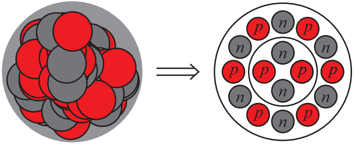

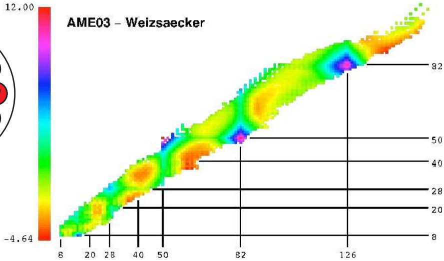

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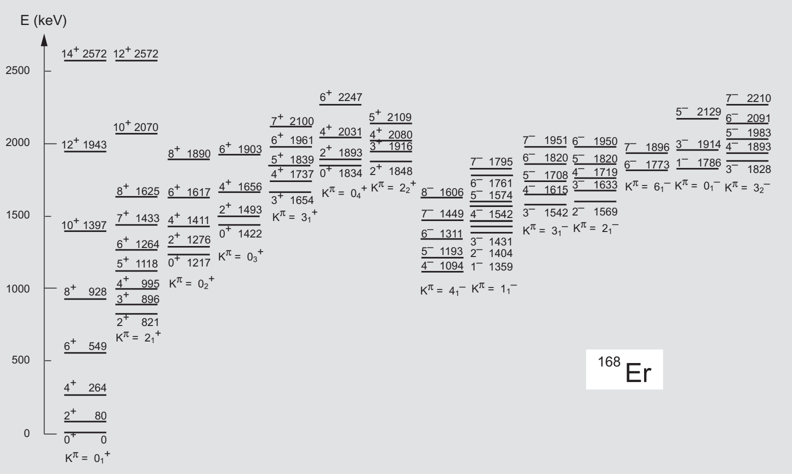

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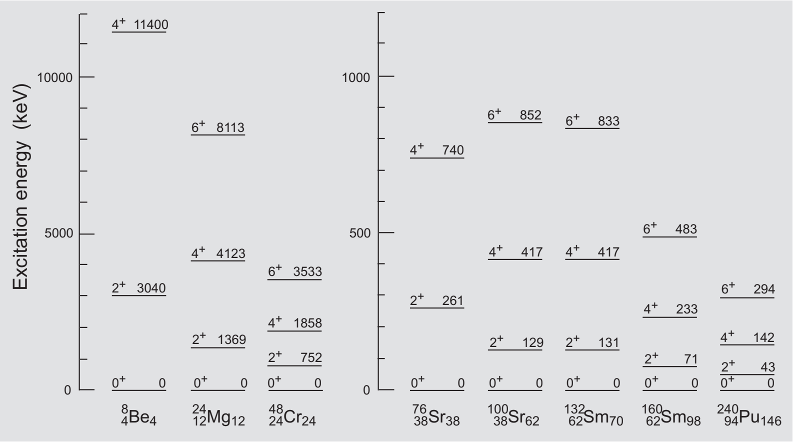

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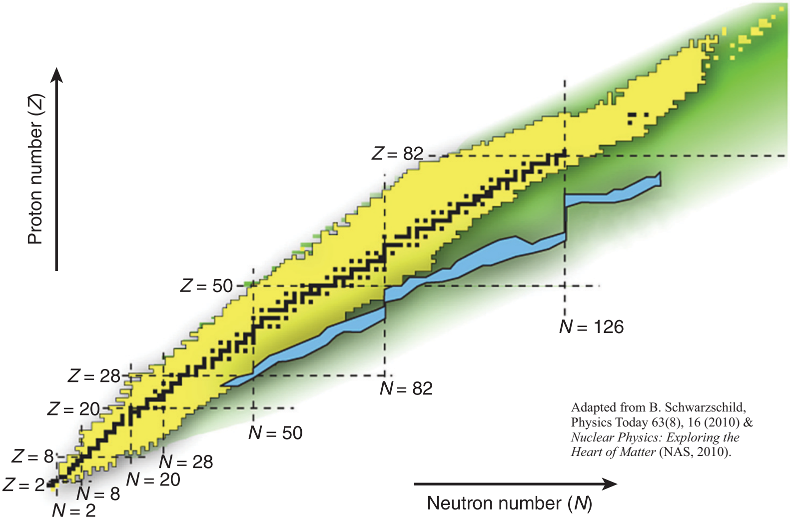

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Observed energy levels for $A = 6$ nuclei

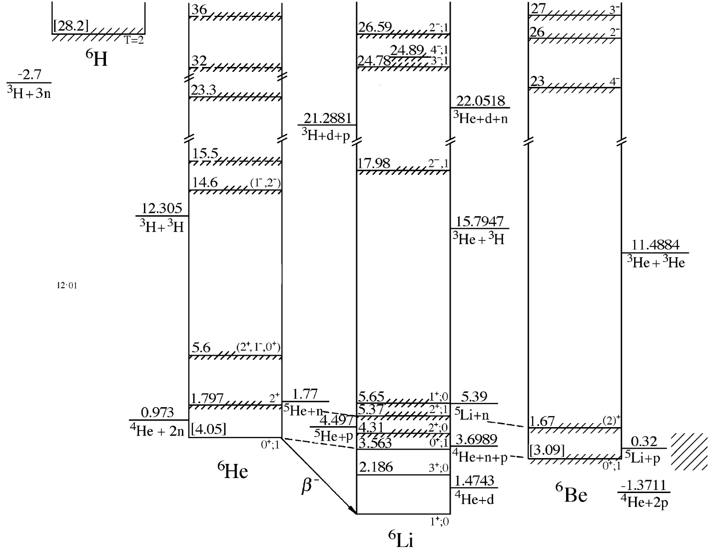

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Figure from D.R. Tilley *et al.*, Nucl. Phys. A **708**, 3 (2002).

The size of halo nuclei

Nuclear halos are very atypical:

- Large matter distribution

  Departing from the R~A$^{1/3}$ dependance

- and different charge and matter radii

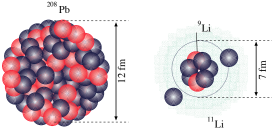
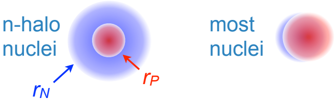
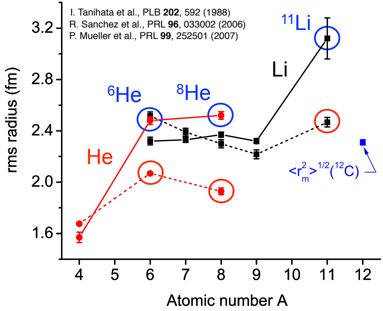


Figure from: B. Jonson, Phys. Rep. **389**, 1 (2004)



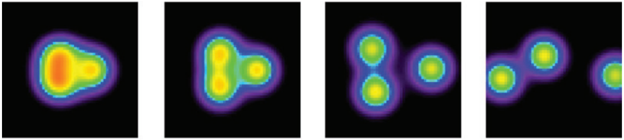


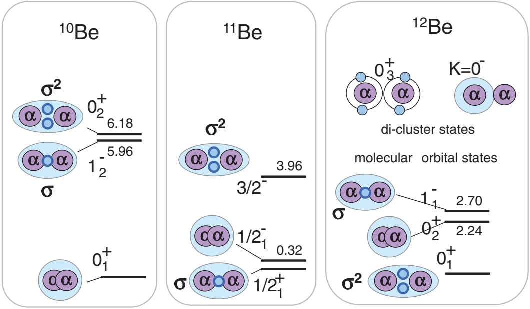
courtesy of M. Brodeur

Cluster structure in light nuclei

0_1^+ 2_1^+ 0_2^+ 0_3^+

^{12}C





Yoshiko Kanada-En'yo, Masaaki Kimura, and Akira Ono, PTEP 2012, 01A202 (2012).

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- Shell model as baseline framework for structure
- Observables and illustrations

Approaches to Nuclear Structure

"The first, the basic approach, is to study the elementary particles, their properties and mutual interaction. Thus one hopes to obtain knowledge of the nuclear forces. If the forces are known, one should, in principle, be able to calculate deductively the properties of individual nuclei. Only after this has been accomplished can one say that one completely understands nuclear structure…

The other approach is that of the experimentalist and consists in obtaining by direct experimentation as many data as possible for individual nuclei. One hopes in this way to find regularities and correlations which give a clue to the structure of the nucleus… The shell model, although proposed by theoreticians, really corresponds to the experimentalist's approach."

–M. Goeppert-Mayer, Nobel Lecture

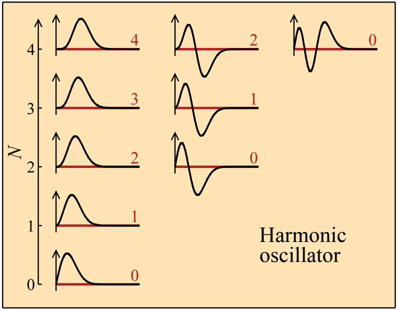
Ab initio approach vs. *phenomenological*

So far, nuclear physics largely phenomenological

Can we describe nuclei from first principles?

courtesy of J. D. Holt

Three-dimensional harmonic oscillator orbitals



One particle in *three* dimensions

$$V(r) = \frac{m\omega^2}{2} r^2 \quad \textit{Central force}$$

$$\Psi(r,\theta,\varphi) = \frac{R_{nl}(r)}{r} Y_{lm}(\theta,\varphi)$$

$$m = -l, -l+1, \ldots, l$$

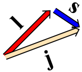
$$N = 2n + l \quad \textit{Major shell}$$

"Oscillator basis" depends upon length parameter b (or on $\hbar\omega$)

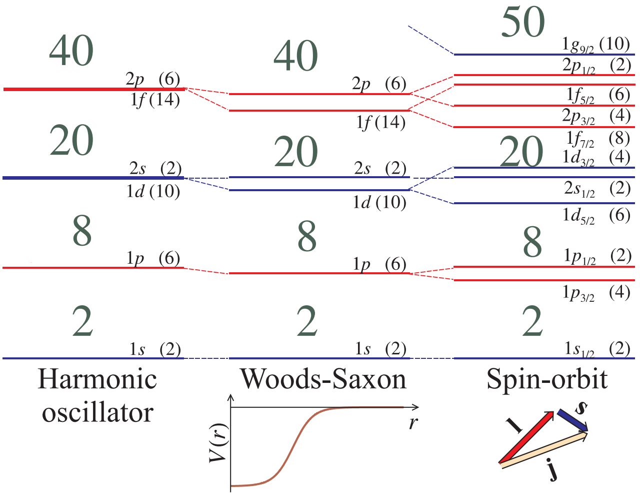
$$R_{nl}(r) \propto (r/b)^{l+1} L_n^{(l+1/2)}[(r/b)^2] e^{-(r/b)^2/2} \qquad b(\hbar\omega) = \frac{(\hbar c)}{[(m_N c^2)(\hbar\omega)]^{1/2}}$$

Couple orbital angular momentum with spin (jj-coupling)

$\Rightarrow$ Single particle basis states $|nljm\rangle$



Shell model orbitals



Spin-orbit interaction $(\mathbf{l}\cdot\mathbf{s})$ $\Rightarrow$ *jj coupling scheme*

The many-particle Hilbert space

For a system of *distinguishable particles*, the Hilbert space consists of all *linear combinations* of *direct products* of *single particle states*.

SIMPLE EXAMPLE: 2 particles, in 2 states ($|\downarrow\rangle$ and $|\uparrow\rangle$)

$$|\Psi^{(2)}\rangle = a_{\uparrow\uparrow}|\uparrow\rangle_1|\uparrow\rangle_2 + a_{\uparrow\downarrow}|\uparrow\rangle_1|\downarrow\rangle_2 + a_{\downarrow\uparrow}|\downarrow\rangle_1|\uparrow\rangle_2 + a_{\downarrow\downarrow}|\downarrow\rangle_1|\downarrow\rangle_2$$

NUCLEAR PROBLEM: A particles, with single-particle basis states $|nljm\rangle$

$$|\Psi^{(A)}\rangle = \sum_{\substack{n_1 l_1 j_1 m_1 \\ n_2 l_2 j_2 m_2 \\ \cdots}} a_{(n_1 l_1 j_1 m_1)\cdots(n_A l_A j_A m_A)} \, |n_1 l_1 j_1 m_1\rangle_1 |n_2 l_2 j_2 m_2\rangle_2 \cdots |n_A l_A j_A m_A\rangle_A$$

But for *indistinguishable particles* (specifically, *fermions*), only linear combinations *antisymmetric* under interchange of particles are permitted.

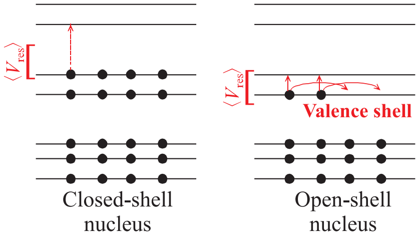
$$|\uparrow\downarrow\rangle = \frac{1}{\sqrt{2}}[|\uparrow\rangle_1|\downarrow\rangle_2 - |\downarrow\rangle_1|\uparrow\rangle_2] \qquad \text{Pauli } \checkmark$$

$$|\Psi^{(A)}\rangle = \sum_{\substack{n_1 l_1 j_1 m_1 \\ n_2 l_2 j_2 m_2 \\ \cdots}} a_{(n_1 l_1 j_1 m_1)\cdots(n_A l_A j_A m_A)} \, |\underbrace{(n_1 l_1 j_1 m_1)(n_2 l_2 j_2 m_2)\cdots}_{\text{Protons}}\underbrace{\cdots(n_A l_A j_A m_A)}_{\text{Neutrons}}\rangle$$

Shell model and collective correlations

$$H = \sum_{i=1}^{A} -\frac{\hbar^2}{2m_i}\nabla_i^2 + \sum_{i,j=1}^{A} V(\mathrm{r}_i - \mathrm{r}_j)$$

$$\Rightarrow \overbrace{\sum_{i=1}^{A}\left[-\frac{\hbar^2}{2m_i}\nabla_i^2 + U(r_i)\right]}^{H_0} + \underbrace{\left[\sum_{i,j=1}^{A} V(\mathrm{r}_i - \mathrm{r}_j) - \sum_{i=1}^{A} U(r_i)\right]}_{V_{\mathrm{res}}}$$



Independent particle model ($H \approx H_0$): Eigenstate approximated as single configuration

Classic shell model ("configuration interaction" calculation):

Many-body problem restricted to *valence* shell

Neglected ("inert") core leads to effective interaction of valence nucleons

Open shell [$\Delta\varepsilon \lesssim \langle V_{\mathrm{res}} \rangle$] permits collective phenomena:

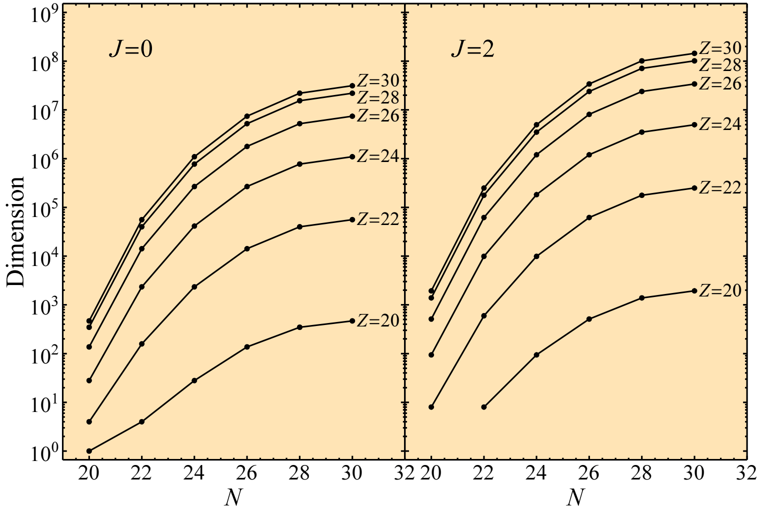
Large number of single-particle configurations energetically accessible

Little energy required for excitation

Single-particle energies in the *pf* shell

Figure courtesy United States Postal Service

Model space dimensions in the *pf* shell



Nuclear structure of ^{56}Ni in the *pf* shell

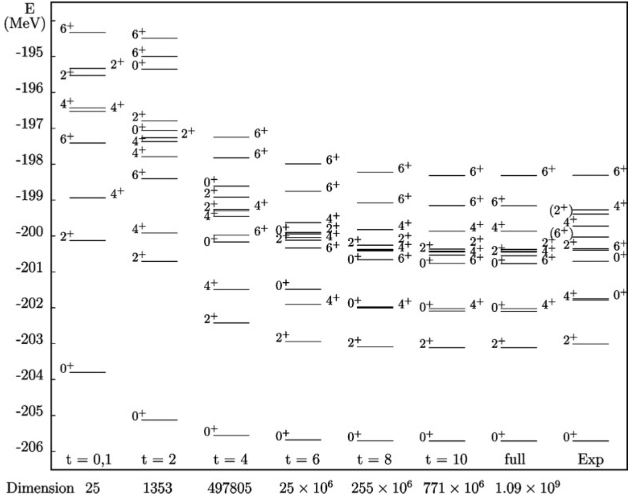


FIG. 1. The evolution of the first three 0^+, 2^+, 4^+, and 6^+ states as a function of the truncation level *t*.

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M. Horoi *et al.*, PRC **73**, 061305(R) (2006). *M*-scheme in *N*-particle *N*-hole truncation.

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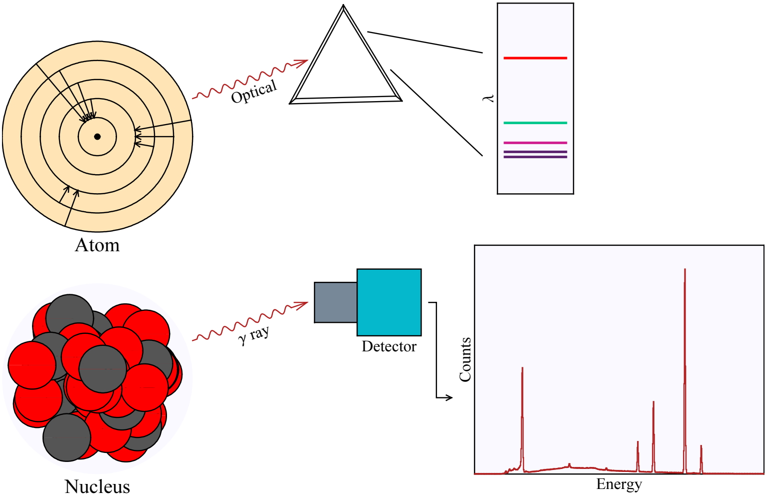
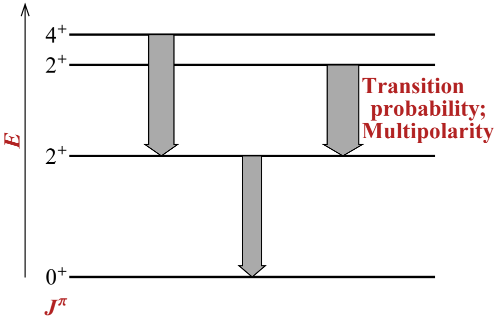
How is nuclear structure studied?





Obtain detailed information on physical structure and excitation phenomena from spectroscopic properties

- Level energies and quantum numbers
- Electromagnetic transition probabilities and multipolarities

Fermi's golden rule $T_{i \to f} \propto |\langle \Psi_f \,|\, \hat{T} \,|\, \Psi_i \rangle|^2$

Electromagnetic probes (e-scattering), α decay, β decay, nucleon transfer reactions, . . .